# A STUDY ON GARCH VOLATILITY PROCESSES IN PRICING DERIVATIVES

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# A Study on GARCH volatility processes in pricing derivatives

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## Abstract

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In this thesis the GARCH models are applied to evaluate financial options and futures. In the first application, the GARCH models in parsimonious form are studied for pricing the S&P500 options. Unlike previous studies that focus on developed formulation, the results indicate that simplified models provide effective performance and it is the simple GARCH model that yields the least valuation error. To our consideration, examining model possessing simplification is of practical importance because model estimation becomes readily accessible through available econometric software, which circumvent programming barriers in implementing alternative one's own pricing methods.

The second application consider the component GARCH models for currency option pricing. The valuation results favour the component formulations particularly in the pricing of long term contracts. Volatility modelling results indicate that the return-volatility relationship is symmetric in the long run, but over the short term asymmetry also arises in the EURUSD and GBPUSD exchange rates.

The third application evaluates canola futures in Canada in relation to spot market price. Results confirm the cointegrating relationship with threshold

corresponding to transaction and adjustment cost. And it is the futures market that adjusts actively to price disparities but in the meantime there is volatility spillover from futures to the spot market.

Overall, our empirical assessments indicate the importance of the time varying volatility and the improvements achieved in option pricing and futures evaluation. We believe the present study's analysis provides useful suggestions and further guidance to practitioners and investors for the pricing and trading in the equity and foreign exchange markets, also to the market agents to better evaluate price uncertainty in order to guard against adverse price changes.

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# Chapter 1. Introduction

By comparing market derivatives prices with those derived based on GARCH theory, this thesis studies the effectiveness of time-varying volatility representations to fully characterise the volatility evolution. The class of GARCH (generalised autoregressive conditional heteroscadesticity) models are recruited to measure and describe the time variation of volatility over time. And the effectiveness of these GARCH models are assessed according to their performance in evaluating derivatives used in the stock, foreign exchange, and commodity market.

In the valuation of derivatives, financial options and futures are considered, which are two of the commonest derivatives used for risk management and have many natures in common. As an option offers its holder the right for the underlying asset buying and selling, a futures contract fulfil its holder the obligation to have the transaction, given any underlying assets entitled to be sold or purchased at a prespecified price. Furthermore, both derivatives have a gearing nature which is inherent in their functions, through the premium of the options and margin paid for the futures. With this small proportion relative to the underlying asset price, the derivatives holder could obtain extra proceeds when the underlying price moves at the right direction, but also raises proportionally greater losses in the case of adverse movements.

## 1.1 Background

The family of GARCH models has its origins in the ARCH (autoregressive conditional heteroscadesticity) model proposed in Engle (1982). Bollerslev (1986) further generalises the ARCH model by introducing the lagged conditional variance into the volatility process representation. The term 'autoregressive' refers to the regression on lagged variables of conditional volatilities and innovations, while the term conditional heteroscedasticity refers to the time varying volatility dependent on its past values. With the original GARCH framework as the cornerstone, in this thesis we study further at three distinct aspects of financial time series that are representable within the GARCH family of models. These are (i) volatility asymmetry, (ii) volatility persistence, and (iii) volatility multidimensionality. We demonstrate that although all three aspects are not necessarily applicable for every financial time series, at least one exists for the price series considered here and that the more comprehensive representation which provides an improved volatility measurement is of practical importance in the derivatives valuation.

Volatility asymmetry is commonly known as the leverage effect in stock markets, because a negative return experienced in stock price results in a more pronounced volatility increase than a positive return (Black 1976). This stylised fact in stock market is fundamentally due to a stock price decrease causing a rise in the debt-equity (leverage) ratio, which is an indicator of firm riskiness and could raise an accompanying volatility increase. In the commodity market, however, an inverse leverage effect could arise since any commodity price increase is deemed as bad news in the markets and raises price volatility further than any negative returns (Alexander 2008a). Finally, the foreign exchange market exhibits more complicated stylised fact with symmetry of return volatility relationship in the long run and asymmetric relationship over the short time horizon. This is because that in the long run a positive return in one currency entails a negative return for the other, but in the short run any behaviours from the noise traders and usage of foreign exchange derivatives could cause an asymmetric return volatility relationship.

Besides the volatility asymmetry, another stylised fact we study is volatility persistence in the financial market. Although conditional volatilities could evolve higher and lower at times over the short run, over the long time horizon the forecasted volatilities are expected to converge to a normal value (Engle and Lee 1999; Christoffersen et al. 2008). The implied volatilities extracted from the traded options price are typically observed as consistent with this mean reversion fact. By examining the implied volatilities over the short and long term to expirations, different mean-reversion rates are revealed and the long run volatility is always less volatile than the short run volatility, and is closer to the long run average volatility of the underlying asset. Conventional GARCH models fail to describe this effect, because the single volatility equation for these models inadequately differentiate between transitory and persistent volatility behaviour, which are considered to be prevalent in foreign exchange markets due to the variety of economic forces impacting on the exchange rate, such as policy and interest rate announcements on one hand and trade and purchasing power parity on the other. An effective solution for dealing with this limitation, is to specify this stylised fact with additional volatility component, with original volatility component describing the temporary information shocks, and an additional long-run volatility component evolving with a slower mean-reversion rate.

Finally, investigating the volatility behaviour of spot and futures prices for a commodity raises a different set of issues owing to its intrinsic multidimensionality. In comparison to the univariate formulation, the methods for estimating a multivariate GARCH process are computationally more formidable particularly in estimating a fairly large GARCH covariance matrix. According to Bauwens et al. (2006) and Alexander (2008a), the important multivariate GARCH models such as the constant conditional correlation model of Bollerslev et al. (1988), the dynamic conditional correlation model of Engle (2002), and the BEKK class of GARCH models of Engle and Kroner (1995) are considered, and the latter is selected because of its rich parameterization, complexity therefore more conductive estimation. Although the dynamic conditional correlation model may be thought to be more efficient for multiple volatility series due to its scalar correlation representation, the BEKK model's parameterised formulation provides a more specific description on information transmission and volatility spillover across markets.

## 1.2 Derivatives under Study

### 1.2.1 S&P500 option pricing under the GARCH volatility framework

Although the Black-Scholes model may be favoured in pricing the European-style options due to its simplicity and closed-form solution, its assumption of constant volatility unavoidably results in inaccuracy in a real world practice. Instead, we can simulate the risk-neutral option price based on the stochastic volatility process of the estimated GARCH model. Besides overcoming the inherent shortcoming of the Black-Scholes model, the family of GARCH formulations has the versatility to capture other effects and the stylised facts to yield more accurate solutions. As an illustration, Christoffersen and Jacobs

(2004) and Hsieh and Ritchken (2005) recommend a non-linear form amongst the asymmetric GARCH models, Barone-Adesi et al. (2008) suggest a threshold GARCH model due to the flexibility, while Stentoft (2005), Stentoft (2008), and Chiang and Huang (2011) advise the exponential GARCH model for its optimal performance in their valuation exercise.

However, the developed option pricing models with further characterised volatility evolution and relaxed model assumptions are often accompanied by the advanced model formulation and complicated numerical methods, which unavoidably impede these models' implementation and raise difficulties to practitioners in examining robustness. To our consideration, a practical model should be as simple as possible, but not simpler than the form retaining necessary endogenous elements that constitute volatility. Following this spirit, we focus on the gap amongst existent empirical inquiries by examining these most parsimonious models in their option pricing performance. And we attempt to answer the question if these simplified GARCH models remain effective in option pricing.

We identify the most effective option pricing model based on its accuracy in simulating the option price, through considering a range of simplified formulations having the potential to accommodate the stylistic nature of the S&P500 index and its volatility evolution. Two constant volatility models are used as benchmarks: (i) the Black-Scholes model and (ii) the Gram-Charlier model. The latter is an important addition, since by incorporating skewness and kurtosis, it addresses market-specific stylised fact such as fat tails in stock and commodity markets. We start from the premise that an option pricing model based on a time-varying volatility should yield an improvement over any constant volatility models, but invoke parsimony and option pricing bias in making our final decision.

Our empirical results suggest that the maximum likelihood estimation for the S&P500 historical price returns support the GARCH models with a leverage effect. Further in agreement with Hardle and Hafner (2000) and Christoffersen and Jacobs (2004), the option valuation results suggest that the leverage GARCH models continue to have a satisfactory performance. However, the more important result is that by aggregating the overall valuation results we find the more parsimonious form ordinary GARCH model without a leverage effect formulation outperforms any other models in our tournament. This finding is very much new and contrasts with most previous empirical results. But in a related paper by Chiang and Huang (2011), they notice that a simple GARCH based form for pricing options performs more effectively during an upturn economy.

Overall, this study contributes to the inchoate literature on the use of parsimonious GARCH models in the valuation of financial options and the development of an ideal model for derivatives pricing under stochastic volatility. To the best of our knowledge, none of the existent studies examine these models at their most parsimonious form. However, the parsimonious form is of practical importance because one could readily obtain these estimated models through any available statistical softwares, rather than spending extra effort for the model coding for alternative one’s owned models, which is a considerable ease when applying these models in practice.

Finally, the present empirical inquiry contributes to the literature for proposing a methodological approach which addresses previously ignored shortcomings such as the potential structural break caused variable model estimates, the inconsistent conclusion on model performance, the identification of suitable and consistent benchmark models, and the selection of appropriate loss function in gauging valuation errors. We believe that by conducting a comparative analysis based on our amended methodology could provide more reliable conclusions on the effectiveness of given parsimonious GARCH option pricing models.

### 1.2.2 Foreign exchange option pricing with GARCH models

The foreign exchange market constitutes the largest share in the global financial market[1]. In view of the increasing need to the management of exchange risk exposure, the search for the more effective approach in pricing foreign exchange derivatives deserves further attention. Therefore after the study of S&P500 stock option valuation we further investigate the foreign exchange market in order to have a broader understanding of the derivatives used in these financial markets.

Despite extensive research, existent findings on the return volatility relationship in foreign exchange markets remain inconclusive. Studies such as Wang and Yang (2009) and Leung et al. (2013) claim that the excess asymmetry in the foreign exchange markets is due to the presence of contrarian and herding investors. In contrast, Bollerslev et al. (1992), Andersen et al. (2001), Maya and Gomez (2008) and Wang and Yang (2009) contend more symmetric exchange rate return volatility interrelation. One explanation for this volatility symmetry is that a positive return in one currency always results in a negative for the other.

[1] For further details see the Bank of International Settlements 2016.

Our other research focus is on the volatility persistence in the forex market. The studies of Engle and Lee (1999), and Christoffersen et al. (2008) criticise the potential limitation of the conventional GARCH models with single volatility equation as the representation for the long memory volatility behaviours. They recommend a volatility representation to be composed of both a short-term transitory and more long-term permanent effects, where the former tend to fade quickly while the latter persists much longer. Christoffersen et al. (2008) show the absence of a long term volatility component seemingly resulting in the underperformance of the single volatility GARCH models in pricing the S&P500 option. In reviewing the related literature, unfortunately, existent researchers who evaluate currency option pricing under the GARCH framework, e.g., Duan and Wei (1999), Bollen and Rasiel (2003), Harikumar et al. (2004), Ulusoy and Onbirler (2014), and Bhat and Arekar (2016), all consider the single volatility representation only in their GARCH option pricing model. Further to this line, we attempt to extend existent GARCH type currency option pricing models based on a component setup, with a short run volatility component capturing the transient volatility behaviour while a long run volatility component reflecting its persistence.

Four distinct GARCH formulations, based on leverage and volatility component formulations, are considered for modelling the volatilities of EURUSD, GBPUSD, and GBPEUR exchange rates and for simulating the European-style option price written on each of them. As the benchmark, the continuous time model of Garman and Kohlhagen (1983) with deterministic volatility function is used in this study’s comparative analysis. Given the aforementioned controversies, we endeavour to answer the research questions that (i) if any of the asymmetry effect is significant and (ii) any potential deficiencies of a single volatility representation in pricing currency options and (iii) is there an universality of the model representation in pricing the foreign exchange option. We hold out hypothesis that a component GARCH model outperforms conventional GARCH models in the simulated pricing of currency options because its multiple volatility function provides a more effective representation of any persistence and transitory volatility dynamic.

We believe that our model evaluation and pricing analysis provides useful guidance to practitioners and other market agents who engaged in foreign exchange trading and tends to have a more clear understanding of the fair value of any European style plain vanilla options written on exchange rates. In view of inconclusive findings about the exchange rates return volatilities relationship, our empirical results further specify the foreign exchange return volatility interrelation. Based on the maximum likelihood estimating results, our results do not show any significant presence of long run volatility asymmetry. However, there is short lived volatility asymmetry found in the EURUSD exchange rate. There is also strong support for the component GARCH model reflecting a persistence and transitory volatility dynamic. Second, unlike previous studies that consider GARCH models with the single volatility equation only in pricing the foreign exchange options, our practice makes the first attempt at applying a component GARCH model in the valuation exercise. In our assessment, again the component GARCH model performs most satisfactory but the asymmetric component form outperforms the remaining models in pricing the EURUSD and

GBPEUR options, which suggest that a component GARCH formulation accommodating asymmetry effect is crucial in the pricing of currency options.

### 1.2.3 Commodity futures evaluation with GARCH volatility

Our final application considers the role of the multivariate GARCH model in the pricing of futures commodity contracts. In contrast to previous work evaluating exclusively the spot-futures price relationship of financial assets and commonly known commodities, our study focuses more specifically on the commodity futures of canola which is a primary commodity futures and is most actively traded in Canada.

Previous studies on canola price discovery concentrate on a linear cointegration and error correction framework, which implicitly assumes a time-invariate equilibrium between the spot-future market price with the continuous adjustment to any price deviations. However, this assumption has implicit deficiency in that the price equilibrium relationship varies in the related way that could result in an incorrect inference about the information linkage between the two markets. Balke and Fomby (1997) and Wang and Wu (2013) point out that the presence of any transaction and adjustment cost can prevent economic agents and arbitrage opportunity, since any profits exploiting the arbitrage fail to sufficiently compensate the cost incurred. Arbitrage is only pursued provided that the implied price deviation exceeds some threshold level. Following this spirit, this work aims to fill the research gap in the existent literature by examining the spot futures price relationship based on a threshold cointegration and error correction methodology.

We raise the research questions that (i) in what patterns do the spot-futures adjust to their price disequilibrium and (ii) to what extent do the their price volatilities affect price change to each other. Further this line, we apply a methodology with threshold cointegration and error correction models to the evaluation of the canola spot-futures price relationship, and a sequence of the GARCH-style models is further applied to investigate any information uncertainty from price changes. The current study focuses on the BEKK type GARCH model because of its rich parameterization enablling one to better identify the presence of volatility transmission and spillover.

Our empirical results confirm the cointegration between the canola spot and futures price movement with adjustment taking place in the futures market to restore price equilibrium. However, the presence of threshold over the cointegration process suggests that arbitrage activity is not continuous and the price adjustment process only occurs when threshold is exceeded. And these adjustments constitute to the majority of the cointegrating process which evolves outside the no-arbitrage band. Moreover, our results from the second moment of price movement indicate that there is volatility spillover from futures to the spot price, which suggests that the large amount futures trading unduly affect the subsequent expectation of spot price movements.

Overall, the study advances our understanding of the role of threshold in the equilibrium adjustment process for canola spot and futures prices, and reveals additional information in respect of the subsequent movements of futures and spot price change. We believe our empirical finding is important as it enables investors to have more effective management to the exposure from any adverse spot market price movements. Furthermore, our analysis on the price volatility between the spot and futures market of canola reveals more knowledge concerning price change uncertainty between the two markets. For further research it can be fruitful to investigate effective risk managing strategies as our volatility analysis sheds light on the effective risk-minimization hedging strategies of the futures market, which is another major function of the futures market.

### 1.3 Thesis Structure

The structure of the remaining thesis is organised as follows: Chapter 2 provides an aggregate review of the literatures of our subject areas. Chapter 3 examines the S&P500 index volatilities and calculates the predicted S&P500 option price based on the estimated GARCH volatility process. We also examine the stylised facts of S&P500 index volatilities, focusing particularly on the leverage effect that has a direct impact on the return-volatility relationship. With the estimated GARCH parameters we then calculate the model price and examine its difference to the market option price. In Chapter 4 we extend the GARCH models to include the long run volatility component, and empirically investigate the benefit that the long term volatility dynamic might have in pricing the foreign exchange options. It is the objective of both Chapter 3 and 4 to examine how effective the GARCH volatility models are for the option price valuation. Lastly, Chapter 5 evaluates the Canadian canola futures price in relation to its underlying spot market when price equilibrium has non-linearity and when there is multidimensionality of the volatility process. We also consider the economic underpinnings of the threshold measure of this process, together with the role that volatility could have in the spot futures markets association. Chapter 6 concludes with a summary of the main findings and important contributions to the literature.

## Chapter 2. Literature review

In this thesis we study the financial options and futures in the presence of GARCH volatility process. We study these derivatives based on respectively the stock market, currency market, and commodity market, to have a broader understanding of these derivatives and their usage in practice. When studying the financial options we examine their values based on a stochastic volatility process as it gives more representative formulation of price change in practice. Therefore the stochastic option pricing models based on a GARCH framework are studied and reviewed due to the great success of GARCH models achieved in modelling the stochastic volatilities. We evaluate not only these models with advances but also their efficiency during the implementation. Furthermore, apart from the applied GARCH models we also consider alternative formulations which have potentials in matching the stylised facts exhibits from the underlying asset markets. Lastly, we review the studies of the financial futures in relation to its underlying spot asset price subject to their price level, price change, and price volatility. The GARCH models also account for an important aspect during the evaluation as it provides measurement for information uncertainties between the spot and futures price interaction.

This chapter is organised as follows. In section 2.1 we study the GARCH type stock option pricing models in the respects of the models with original contribution, models with closed-form, models involved with exogenous factors, models based on alternative distributions, models with relaxed market completeness assumptions, and the models with more realistic pricing kernels. Section 2.2. reviews the GARCH type currency option pricing models. In this section we focus more on the stylised facts in the foreign exchange rate volatilities and existent volatility representations provided by the GARCH models, and we consider the further generalisation of existent GARCH models in the foreign exchange option pricing. Finally in section 2.3 we review the related studies on financial futures and its information role during the price discovery and volatility spillover as well as transmission. Lastly, section 2.4 provides a summary of our review.

## 2.1 Stock option pricing with GARCH models

The cornerstones of option pricing theories are attributed to the seminal works of Black and Scholes (1973) and Merton (1973), who made breakthrough in the theory of option valuation. Of their models, several ideal conditions are assumed during model derivation, including the Brownian motion of underlying price movement, log-normal price distribution, constant interest rate, frictionless security markets, and instantaneously constant volatility. Among these, the assumption being criticised mostly would be the instantaneously constant volatility used for computing the option price over term to expiration (Fouque et al. 2000; Bates 2003; Alexander 2008b; Christoffersen et al. 2012c). Subsequent studies, particularly continuous time models such as Hull and White (1987), Heston (1993), Bakshi et al. (1997), Dumas et al. (1998), generalise this constant volatility assumption and consider the use of updated implied volatilities over time in option pricing.

Unfortunately, even though these studies generalise the instantaneously constant volatility condition of Black-Scholes model with greater precision, empirically implementing these models remain computationally cumbersome. Under the continuous time framework, one needs to filter a continuous volatility variable from discrete price observations, and infer implied volatility subject to each striking price and expiration date from lengthy option price series. In comparison, a GARCH model with discrete time formulation provides a more natural description in respect of the discrete observations based on the discontinuously recorded market data.

Duan (1995) proposes the first contribution of applying a GARCH model for the European style option valuation, in which a pricing kernel termed LRNVR (locally risk neutralised valuation relationship) is proposed for deriving a GARCH type option pricing formula. Subsequent studies by Lehar et al. (2002) and Huang et al. (2011) compare the Duan's (1995) non-linear GARCH model with the continuous time model of Hull and White (1987), and conclude that the GARCH model is preferred.

Schmitt (1996) considers the GARCH model in exponential form of Nelson (1991) and suggests that the EGARCH better explain the implied volatility pattern. Yung and Zhang (2003) compare the EGARCH with the Practitioner's Black-Scholes model of Dumas et al. (1998), and posit that the EGARCH model dominates both in-sample and out-of-sample assessments. Hardle and Hafner (2000) examine a GARCH model with threshold and suggest their amended threshold GARCH model can produce less valuation bias. Christoffersen and Jacobs (2004) evaluate the family GARCH model of Hentschel (1995) with its nested GARCH specifications subject to their option pricing performance. Their results favour a non-linear and asymmetric GARCH model than other more parameterised models. Lastly, for GARCH models with jumps in price and volatility can be found in Duan et al. (2006), Christoffersen et al. (2012b), and Durham et al. (2015). Duan et al. (2006) proposes the first GARCH-in-Jump model. Christoffersen et al. (2010a) study GARCH models when return distribution is influenced by dynamic volatility and jump intensity. Durham et al. (2015) further refines the discrete model of Christoffersen et al. (2012b) by aligning their filter and estimator in accordance with estimated model parameters.

Besides the aforementioned GARCH models requiring numerical methods for computing option price, Heston and Nandi (2000) developed another important type of model which embodies an almost closed-form option valuation solution. Subsequent empirical studies following Heston and Nandi (2000) include Hsieh and Ritchken (2005), Su et al. (2010), Huang et al. (2017a), and Huang et al. (2017b). Hsieh and Ritchken (2005) posit that the analytical solution of the closed form model would comes at a cost of realism owing to its over-restricted volatility equation. Su et al. (2010) suggest that the Heston and Nandi model yield less valuation errors than the Practitioner's Black Scholes model in the FTSE100 index option valuation assessment. Huang et al. (2017a) apply Heston and Nandi GARCH model in valuing the stock options listed in the American stock exchange and suggest that liquidity has no significant impact to the option valuation performance. Huang et al. (2017b) study the realised volatility GARCH model with the edgeworth expansion used for the option pricing formula. Their results suggest that applying high frequency data as the measurement for volatility could provide improved valuation performance and suggest that it is favorable to feed the realised volatility through the Heston and Nandi model to have further efficient and accurate valuation performance.

Christoffersen et al. (2008) further develops the closed-form model of Heston and Nandi (2000) onto a component setup in light of the component GARCH model given in Engle and Lee (1999). This model embodies both the short and the long run volatility equations to respectively describe the transitory and persistence volatility dynamic. Dziubinski (2011) studies the component GARCH model of Christoffersen et al. (2008), and argues that the analytical solution of the non-linear component GARCH structure may cause negative evolution of conditional variance, and proposes an amended model with more parsimonious equation. A more explicit asssessment of the component GARCH model can be found in Christoffersen et al (2012), in which the component GARCH models with/without the analytical solution are compared. The results suggest that a component model without affined restrictions from an analytical form yields less valuation bias, though computation cost is unavoidably more demanding.

For GARCH option models that investigate the exogenous factors such as overall economy condition and market systematic risk level and attempt to embody these element in model formulation, related practice can be found in Chiang and Huang (2011), Wang et al. (2012), Kanniainen et al. (2014), Papantonis (2016), Tang and Diao (2017), and Wang et al. (2017b). Chiang and Huang (2011) investigate the asymmetric GARCH models and GARCH-in-jump models performance from the market momentum perspective. They conclude that an exponential GARCH model give more effective performance in a downturn economy. On the contrary, a simple GARCH model gives better performance during the economy upturn. Wang et al. (2012) aggregate a capital asset pricing model with the GARCH model for option valuation where system risk is taken into consideration, and found that a volatility smile with twist which suggest that systematic risk could affect option valuation results. Kanniainen et al (2014) and Papantonis (2016) study the GARCH option pricing models when VIX (the volatility index) is taken into account during volatility valuation. Their results suggest that the jointly estimated model parameters could improve efficiency in the option valuing exercise. Wang et al (2017) study GARCH volatility process which embody the VIX variable also in predicting the Taiwanese stock index options. Results indicate that GARCH with VIX volatility process generate less valuation errors and common GARCH volatility and historical volatilities. Lastly, Tang and Diao (2017) consider the use of a hidden Markov model with two states and the GARCH volatility process jointly to facilitate the Black-Scholes model's option pricing performance. Their results suggest that it is favourable to consider this procedure than the traditional GARCH volatility process.

The GARCH models considering alternative distributions rather than Gaussian normal distribution are proposed in Kaminski (2013) , Liu et al. (2015), and Rombouts and Stentoft (2015a). Kaminski (2013) carries out an WIG20 options pricing exercise and suggests the GARCH option pricing model with a student’s *t*-distribution. Liu et al. (2015) study the GARCH model when Hansen’s skewed-*t* distribution is applied with derivation of related moment generating function. Empirical results suggest that their model generally produce better valuation results than the Black-Scholes model. Rombouts and Stentoft (2015a) propose an asymmetric GARCH in mean models with normal mixture distributions in that the innovation term is evolving with a combinations of K densities. In their model confidence set test, they suggest that the component GARCH model and GARCH-in-jump model produce larger valuation errors than their normal mixture distribution model.

Other developments on the GARCH option pricing models that attempt to relax the original model assumption in the aspect of market completeness can be found in Barone-Adesi et al. (2008), Byun and Min (2013), Christoffersen et al. (2013), Rombouts and Stentoft (2014), Rombouts and Stentoft (2015b), and Simonato and Stentoft (2015). Barone-Adesi et al (2008) study the GARCH option model performance under incomplete market and considering the filtered historical innovations. They also suggest that the flexible change of pricing measures could induce the accuracy in the option valuation. Byun and Min (2013) resolve the overfitting problem of Barone-Adesi et al (2008) and induce different descriptions of conditional volatility dynamic under the physical and risk neutral measures. Simonato and Stentoft (2015) examine the difference between the equilibrium model and no-arbitrage assumption models with the Johnson skewed and luptokurtic distribution applied. They found that there is no significant valuation bias for both types of pricing frameworks, but the equilibrium model is preferred due to the valuation efficiency.

Lastly, the GARCH option pricing studies with particular focus on pricing kernels are proposed in Christoffersen et al. (2013), Ryu et al. (2015), Babaoglu et al. (2017), Badescu et al. (2015), and Badescu et al. (2017). Christoffersen et al. (2013) develop the GARCH model of Heston and Nandi (2000) by specifying the more general monotonic valuation kernels which is a function of the return and return variance also. Their results indicate that the new pricing kernel better reconcile the empirical distribution of underlying returns and distribution implied from the option price. Byun et al. (2015) examine the variance premium of the GARCH-in-jump model with the pricing kernels of Christoffersen et al (2013). They suggest that the model incorporates the jump risk premium and variance premium give out performance in their valuation exercise. Ryu et al. (2015) propose an implied pricing kernel method where model parameters under physical probability measure are implied by the GARCH option models. They posit that their amended implied kernels produce slightly improved pricing performance particularly on the out-of-the-money options. They claim that their model is more advantageous in the valuation of options with insufficent past pricing information and speculative and volatile markets. Babaoglu et al. (2017) study the u-shaped non-monotonic pricing kernel and emphasize its economic significance in option fitting. Badescu et al. (2015) apply the extended Girsanov principle and the conditional Esscher transform as pricing kernel candidates, which further complete the LRNVR relationship defined in Duan (1995). Badescu et al. (2017) investigates the valuation and convergence performance of non-normal distributed GARCH models with risk-neutralization based on a variance-dependent exponential linear pricing kernel, and the market price for risk has stochastic description. Their empirical results indicate that the non-linear GARCH model with Gaussian mixture distribution performs best while a GARCH model with Gaussian distribution consistently underperforms.

### 2.1.1 The critiques from the review

With reviewing the aforementioned literature on stock option pricing with the GARCH models, following concerns are raised which need to be addressed prior to the most efficient model contender being chosen. These are (i) the short of consistency of the model formulation amongst previous empirical enquiries, (ii) the model mis-specification particularly due to the structural break during the parameter estimation, (iii) the identification of the benchmark for model comparison, and (iv) the inconsistent loss function use in the measurement of the valuation errors.

With regard to the first issue, the existent findings on the most efficient GARCH formulation in valuing stock index options appears to be conflicting. In Christoffersen and Jacobs (2004a), a GARCH representation with merely a non-linear structure with a leverage effect parameter is suggested in the option valuation. In Hsieh and Ritchken (2005), the non-linear GARCH representation is shown of having less valuation errors than the closed form GARCH model of Heston and Nandi (2000). In Barone-Adesi et al (2008) the GARCH model with the threshold effect is advised rather than the GARCH model that contains the jump effect, which is attributed to the flexible formulation of the threshold GARCH model in representing the leverage effect. Concerning the papers of Stentoft (2005) and Stentoft (2008), who study the American options with GARCH volatility process, and Chiang and Huang (2011) who examine common asymmetric GARCH models and GARCH-in-jump model performance, it is suggested that the EGARCH model rather than other models due to its valuation performance. Although these practice all show the significant developments in the GARCH option pricing models study, the present study endeavours to resolve these inconsistencies from previous findings.

Second, this present study examines the significance of any model estimating bias that arise from the heterogeneity of sampled data. Although a sufficient long time series could facilitate the accuracy of model estimating results, the potential structural breaks could cause estimating bias (Wooldridge 1990; Andrews and Ploberger 1994; Hall et al. 2003; Smith 2008; Karanasos et al. 2014). As a consequence, model mis-specification could arise and may undermine the robustness of the volatility modelling result. In this regard, the potential structural breaks over data series ought to be examined thoroughly prior to any valid conclusion on the best model representation that can be made.

Third, the use of effective benchmark during comparative analysis remain unclear and insufficient. In Duan (1995) and Hardle and Hafner (2000), it is the Black-Scholes model used as benchmark while in Heston and Nandi (2000) the ad hoc Black Scholes model of Dumas et al. (1998) is taken into account. In a further assessment of Hsieh and Ritchken (2005) the Duan's non-linear GARCH model is used as benchmark for gauging performance of their threshold GARCH model. Although the comparison from any benchmark models could yield reasonable conclusion of model performance, frequent and inconsistent use of benchmark models could result in misleading findings. Furthermore, in recognition of the GARCH model's fat tail distribution providing extra characterisation of the underlying returns' riskiness, a continuous time benchmark model allowing for additional skewness and kurtosis would be more analogous with GARCH models in this regard.

Finally, our last critique is labelled on the inconsistent loss function use amongst previous research. Christoffersen and Jacobs (2004a) posit that aligning the loss function during both the parameter estimating stage and option pricing stage could yield about 50% improved accuracy from the Black-Scholes model could be achieved. Unfortunately, for existing findings there is inconsistent loss function use for the measurement of the final valuation results. In Bakshi et al. (1997), Heston and Nandi (2000), Hsieh and Ritchken (2005), and Christoffersen and Bacob (2004b), the absolute difference value between the model and market option price is used as loss function. However, the absolute loss function has deficiency that the excessive weight could be assigned to the in-the-money long expiration options, because of the relative expensive price of these option contracts. In comparison, the practice of Jacquier and Jarrow (2000) compares the model and market option value based on the relative percentage measure. The advantage of this measurement is that the results are more intuitively revealing from a rate-of-return aspect. However, the loss function formulated in this way comes at short of weighting the out-of-the-money short-maturity contracts. In this regard, to guarantee the consistency and fairness of the evaluation analysis, it is important to examine the option price magnitude in order to have the most valid valuation results.

## 2.2 Foreign exchange option pricing with the GARCH models

With the option pricing models developed for pricing the stock options, another important extension is to generalise these models for the pricing of the alternative market derivatives. Garman and Kohlhagen (1983) and Biger and Hull (1983) propose the original models for the currency option valuation, which are developed from the Black-Scholes models with assumptions mostly retained. The first GARCH type model used for pricing the foreign exchange options can be found in Duan and Wei (1999). Under their framework, the procedure of deriving the risk-neutralised GARCH models of Duan (1995) is generalised into a two countries economy therefore for currency options written on any bilateral exchange rates could be valued accordingly.

**Table 2.1 Studies on FX option pricing using GARCH models**

| Authors | Numerical methods | Volatility style consideration | GARCH models being considered | Major Findings | Research Contribution |
|---|---|---|---|---|---|
| Duan and Wei(1999) | Monte Carlo Simulation | GARCH volatility | Non-linear asymmetric GARCH (1,1) model | Simulation results show that the proposed GARCH model adopts empirical properties in the currency option markets | Develop the GARCH framework for the valuation of foreign exchange options. |
| Bollen and Rasiel (2003) | Lattice Method | regime switching volatility, GARCH volatility, Jump-diffusion volatility | Non-linear asymmetric GARCH (1,1) model | The GARCH option model and the Jump-diffusion models outperforms the regime-switching model, while the ad hoc Black-Scholes model underperform other models. | The first comparison being made among the GARCH, regime-switching model, and jump-diffusion model. |
| Posedel(2006), Irena (2009), Aduda JA (2011) | Monte Carlo Simulation | GARCH volatility | standard GARCH(1,1), Non-linear asymmetric GARCH (1,1) | The GARCH model fits the foreign currency distribution better than the Brownian model, no empirical results in terms of option pricing performance | Theoretically discuss the potentials of using GARCH for pricing local currency options, though the trading of local currency options is yet to be introduced. |
| Manzur, Hoque, and Poitras(2010) | Black-Scholes formula | GARCH volatility, implied volatility, intraday volatility. | Autoregressive Model with standard GARCH (1,1) for conditional volatility | The Intraday volatility model outperforms the GARCH volatility and implied volatility models | Illustration of using realised volatility with high frequency data for the currency option pricing. |
| Gozgor and Nokay (2011) | Black-Scholes formula | GARCH volatility, EGARCH volatility, EWMA volatility | standard GARCH (1,1) EGARCH(1,1) models | GARCH (1,1) and EWMA volatility have no significant difference on USDTRY and EURTRY option pricing. The EGARCH model suggests an asymmetric effect on the volatility. | Consider the EWMA volatility for calculating the USDTRY and EURTRY options, the trading of local currency options is yet to be introduced. |
| Harikumar and Boyrie (2004), Ulusoy and Onbirler (2014), Bhat and Arekar (2016) | Monte Carlo Simulation | GARCH volatility; Implied volatility, current volatility | Non-linear GARCH (1,1) model | The overall valuation performance of NGARCH is inferior to the BS. | Empirical evaluation of GARCH option pricing performance of the USDGBP, USDCHF, USDJPY, USDTRY, EURTRY, and USDINR exchange rates options. |

There are a number of subsequent studies valuing Foreign exchange options under the GARCH framework. Table 2.1 summarises these studies according to their methodologies and empirical results. As the table illustrates, Bollen and Rasiel (2003) examine the performance of the regime-switching model, the jump-diffusion model, and the GARCH model. Their results indicate that both the jump model and GARCH model outperform the remaining models. Harikumar et al. (2004) employ both the GARCH models and the Black-Scholes model for pricing the currency options written on Pound Sterling, Japanese Yen, and Swiss Francs, and they find that the Black-Scholes model outperforms the GARCH model. Posedel (2006), Irena (2009), and Aduda (2011), study the GARCH models in theory and suggest that price dynamic under the GARCH volatility process has a better fit to the exchange rate distribution than the Brownian models. Of the more recent studies, Ulusoy and Onbirler (2014) and Bhat and Arekar (2016) apply the non-linear GARCH model to the currency option valuation, and suggest that the Black-Scholes model yields better currency option valuation results.

### 2.2.1 The critiques from the review

Although the aforementioned studies made significant contributions to the FX option models development, two critiques are raised concerning previous papers. Firstly, previous arguments appear controversial with regard to exchange rate return and exchange rate volatility. Carr and Wu (2007), Bakshi et al. (2008), and Leung et al. (2013) posit that it is the volatility asymmetry existing in the foreign exchange market which arises from the noise traders and hedging activities using currency options. This restricts downside exposure but allows potential upside proceeds. On the contrary, Hsieh (1988), Bollerslev et al. (1992), Andersen et al. (2001), Maya and Gomez (2008) suggest that foreign exchange rate and exchange volatility should exhibit a more symmetric relationship. As foreign currency depreciations could result in negative returns to the foreign currency holder, it raises positive proceeds to the domestic currency holders.

Our second criticism concerns the treatment of long term persistence of the foreign exchange volatility in the volatility modelling of previous studies. Existent currency option pricing mostly considers the GARCH models with singular volatility equations. However, a GARCH model that adopts both short and long run volatility characters might better characterise the exchange rate volatility. The supportive arguments for this criticism is that in view of the actively traded foreign exchange markets (Dornbusch 1976), it is observed that over the short time horizon the foreign exchange rate tends to respond more sharply to the arrival of new information. But such price movements are usually accompanied with swift time decay. Over the long time horizon, however, in light of the law of one price and the purchasing power parity, it is the price level differential between two currencies that determines the long term movement of this exchange rate (Kilian and Zha 2002). In this regard, in addition to previous studies considers the GARCH model with only one volatility equation, it would be more conductive of accommodating both short and long run volatility behaviours in foreign exchange volatility modelling and thus facilitate the valuation of the foreign exchange options.

### 2.3 Commodity futures evaluation with GARCH volatility

In addition to the pricing of stock and currency options under the GARCH framework, it is interest of the third empirical inquiry that studies the commodity futures. The futures contract is evaluated in relation to its underlying spot price movement so as to better understand this derivative. In the meantime, the third empirical study examines spot and futures volatility movements under a GARCH framework to further identify the uncertainty of their price changes. In reviewing the literature, we start from related studies based on equity, currency and commodity markets in general, then come specifically to the canola futures market which is the particular focus of our third empirical inquiry.

Related studies evaluating futures contracts include Dwyer et al. (1996), Tse (1999), Zhong et al. (2004), Kayali and Celik (2010), Theissen (2012), Gakhar (2016), and Wang et al. (2017a), which are based on equity markets. These studies have common findings that futures play a leadership role in price discovery and is swifter in impounding new information. Chen and Tsai (2017) examine the determinants of price discovery of U.S. VIX indices and index futures. They found that VIX futures plays a dominant role in responding to the latest news announcements on macro-economic issues in the United States.

In comparison, other studies find the leading information role of spot market over the futures market in price discovery include Gong et al. (2016), Karabiyik et al. (2017), Sakthivel et al. (2017), and Wang et al. (2017a). Gong et al. (2016) suggest that well-established markets like HSI and S&P500 index exhibit a pronounced leading role in reflecting new information. In contrast the HSI index dominates its index futures in price discovery. Karabiyik et al. (2017) examine the stocks of Islamic companies listed among 19 countries, and find that it is the spot market of these stocks that play the more informational role in reflecting the new information arrival. Additionally they suggest a panel error correction model which facilitate analysis when a large amount of time-series is involved. In examining the CSI 300 futures Wang et al. (2017a) identified the leading role of the CSI 300 spot market using high frequency data. The results indicate that the futures price leads spot price in their price discovery and information transmission in about five minute. Sakthivel et al. (2017) investigate the Indian Rupee price and price volatility in relation to the USD, JPY, GBP, and EUR currencies. The results indicate that all exchange rates exhibit long run cointegration. But, it is the spot foreign exchange market that adjusts actively to the disequilibrium with unidirectional volatility spillover on its futures market.

Related researches based on commodities futures include Schroeder and Goodwin (1991), Mamatzakis and Remoundos (2011), Wang and Wu (2013), Arnade and Hoffman (2015), Singhal and Ashra (2015), Adammer et al. (2017), and Raghavendra et al. (2016), whose studies focus on respective commodity assets and identify the spot-futures price cointegration with price adjustments maintaining price equilibrium. Exceptional studies such as Fortenbery and Zapata (1997), who investigate the cheder cheese market, and as Mattos and Garcia (2004), who investigate the thinly traded agricultural commodities in Brazilian futures market, both do not found the equilibrium during the spot and futures price movement. Potential causes are insufficient trading and transaction volume as well as inconsistent cash and futures market regulations.

For more recent studies, Dimpfl et al. (2017) reconsider the price discovery of a range of primary seasonal and non-seasonal agricultural commodities. In their study the refined measure for the contribution in price discovery is applied which investigate different volatility regimes when identifying the information share. They conclude that the VECM approach is valid and the efficient price of agricultural commodity is determined in the spot market over the long run. Quintino et al. (2017) analyse the relationship between the ethanol spot and futures prices in Brazil in which the Engle and Granger cointegration approach is employed and the information share is identified for each market based on the Hasbrouck method. The results indicate that it’s the cash market that plays the dominant role in price discovery. The underlying cause is that its ethanol mills distribute the wholesale ethanol which affects the formation of market pools. Furthermore, in that ethanol market, only a few distributors control the market shares.

The studies on canola futures, which is the particular focus of the third empirical study in this thesis, include Khoury and Yourougou (1991), Sephton (1992); Sephton (1993), Brockman and Tse (1995), Carter (1996), and Adammer et al. (2017), to date. In Khoury and Yourougou (1991) and Brockman and Tse (1995), the interrelationship of canola spot-futures price is examined and it is found that traders with market-wide information tends to enter the futures market first. They emphasize that the futures market represents the expected spot price in the future and facilitates the forecasting of spot price behaviour. In comparison, Carter (1996) argues that the canola’s futures price remains inefficient as its spot and futures price lack convergence over the delivery month. Also the rail transport regulation could impact the canola market efficiency between the canola spot and futures markets, since the rail car regulation affect the futures and street price of canola.

Sephton (1992) examines the information link between the exchange rate and barley, canola, and wheat price traded in Winnipeg commodity exchange. Results indicate the currency depreciation Granger causes the inflation of commodity price over the short term, but such an interrelation lasts in the long run. In subsequent research of Sephton (1993), a multivariate GARCH model is applied to modelling same commodities' volatilities and the result indicates that the GARCH hedge ratios appear more reliable than the optimal hedge results obtained from the traditional regression approach. Adämmer et al. (2017) evaluate the price and volatility transmission between the north American and European agricultural commodities, and suggest that it is US markets lead price transmission and volatility spillover and actively that adjusts to the price disparity across markets.

### 2.3.1 The critiques from the review

The objective of the third empirical inquiry is to evaluate the relationship between canola spot and futures price in Canada., in the presence of a non-linear threshold cointegration and time varying volatility. For previous studies examining the canola market, they consider the conventional approaches such as the Engle and Granger approach and Johansen procedure which assume that the spot-futures price relationship is time-invariant. However, in the presence of transaction and adjustment cost as well as any structural changes of the market conditions, a time-invariant long run price relationship appears less likely. For any price disequilibrium that arise across markets, arbitragers should only step in when the proceeds from arbitrage exceed any potential transaction and adjustment costs (Balke and Fomby 1997; Wang and Wu 2013). And related expenses should be measured as a no-arbitrage band. In the case of a price disequilibrium occurrence but less than any transaction cost, the random walk of the price movement is expected. In this regard, for any empirical investigation evaluating the spot-futures price relationship, the non-linearity representation adopting threshold effect should be considered to ensure the robust conclusion to be made.

### 2.4 Summary

In sum, the literature review of this thesis focus respectively on three aspects of the related studies on derivatives, which are the stock and currency options pricing and commodity futures evaluation with the GARCH volatility process.

Following research gaps are identified from the reviewed literature, which are (i) identify the efficient GARCH models possessing parsimony and essential stylised volatility for the S&P500 index option valuation, (ii) extend the existent singular GARCH models and apply the component formulation for the foreign exchange options pricing, and (iii) evaluate the Canadian canola futures particularly its information role in relation to its spot market. This thesis aims to carry out empirical inquiries based on these research gaps and make further contributions upon previous studies.

## Chapter 3

# Simulating S&P500 index options based on GARCH estimators☆

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## Abstract

European options on the S&P500 index are priced from a range of popularly used GARCH models because of time-varying volatilities using a Monte Carlo simulation. The present study's results confirm the importance of non-linear asymmetric structure to enhance GARCH models' ability in pricing S&P500 options. However, the least valuation errors from the S&P500 call option provide evidence which supports the simple GARCH model for option valuation. Moreover, after the simple GARCH and the NA-GARCH models, the Gram-Charlier model accounting for the third and fourth moments also exhibit accurate performance.



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## 3.1. Introduction

This chapter aims to evaluate the performance of a range of simplified GARCH formulations on S&P 500 (Standard and Poor 500) option pricing. Accurately evaluating financial options is of practical importance for any practitioner and market participant in the derivatives market. As volatility directly measures risk magnitude and characterises its movement over time, it is expected the option models accommodating the volatility variation provide a more effective measurement of the forecasted option price. We consider the GARCH style models in recognition of their advantages of describing the time varying natures of the volatility dynamic. For the selected GARCH models, we intend to examine the relative performance of each parsimonious GARCH model in order to identify a more accessible and effective option pricing procedure.

As our review of the literature discussed in Section 2.1, we aim to address four crucial issues from previous researches before a definitive conclusion on the choice of the best performing model contender to be made. These are (i) the lack of model consistency by the various researchers, (ii) the existence of model mis-specification and structural breaks, (iii) the identification of an appropriate benchmark, and (iv) the choice of a single effective loss function.

With respect to the first issue, the reported findings on the most appropriate GARCH structure in pricing equity index options remains conflicting. Christoffersen and Jacobs (2004a) employ a family of GARCH models studied in Hentschel (1995) and use these models for evaluating the S&P500 options. They conclude that a GARCH model with a non-linear structure and leverage component is sufficient in modelling the volatility of a financial series. Hsieh and Ritchken (2005) compare the non-linear GARCH model of Duan (1995) and the closed-form GARCH of Heston and Nandi (2000), and conclude that Duan's non-linear GARCH model has superior performance. Barone-Adesi et al. (2008) examine the relative performance of the GJR-GARCH and GARCH-jump models, and conclude that the former is the preferred formulation because of its flexibility in capturing the leverage effect. Stentoft (2005) and Stentoft (2008) examine the American option pricing performance among various GARCH models and suggest that an exponential GARCH model yields the least pricing bias. While building on these developments in formulating the GARCH option pricing model, we also endeavour to resolve the inconsistencies in their findings.

Second, we examine the robustness of estimated parameters and potential pricing bias in the presence of structural break over the data sample. An increase in the data span leads to a lower variance estimator but the estimator for the mean level may become biased due to the existence of structural breaks introduced from having such a long data set. Previous studies, such as Wooldridge (1990), Andrews and Ploberger (1994), Hall et al. (2003), Smith (2008), and Karanasos et al. (2014), argue that the effect of structural break on long span data series may lead to unstable model parameters and therefore cause model mis-specification. This undermines the models' efficiency when assessing the volatility dynamic over time. In light of this, the possibility of structural breaks ought to be investigated thoroughly before robust conclusions on the best performing model contender can be made.

Third, the appropriate benchmarks for comparing models remain unclear. Duan (1995), and Hardle and Hafner (2000) use the original Black-Scholes model as benchmark while Heston and Nandi (2000) use the Black-Scholes model modified by a volatility curve fitting technique of Dumas et al. (1998). The study by Hsieh and Ritchken (2005) adopts the standard non-linear GARCH model as benchmark to assess their threshold GARCH (TGARCH) model's performance. Although the Black-Scholes is an ideal benchmark due to its heavy use in practice, the model can be easily improved by accommodating the third and fourth moment of the underlying price distribution. Also, a continuous model that takes into account skewness and kurtosis will be more analogous with the GARCH model that accommodates underlying asset prices' fat-tail distribution. Because of this, we use as well the pricing model based on the Gram-Charlier expansion as a benchmark in addition to the Black-Scholes model.

Finally, an inconsistent use of loss functions arises when researchers are examining the valuation errors from the option models. To ensure consistency and fairness, a single effective loss function for assessing the option valuation errors for each model formulation has to be applied, otherwise conclusions may become distorted. Christoffersen and Jacobs (2004a) observe that aligning the loss function for both the estimation and evaluation stage leads to an over 50% improvement when using the Black-Scholes model. Bakshi et al. (1997), Heston and Nandi (2000), Hsieh and Ritchken (2005), and Christoffersen and Jacobs (2004a) use the absolute loss function based on the absolute difference between the model and market option price. However, its drawback is the excessive weight given to in-the-money long maturity options because of the relatively higher price of these contract. Instead, other researchers such as Jacquier and Jarrow (2000) use the relative loss function, which compares the difference with the actual price. Although this is reasonable from a rate-of-return perspective, it does suffer the disadvantage of excessively weighting out-of-the-money short-maturity contracts. On balance, we select a relative measure to gauge valuation errors in our search for the best performing model.

In line with the constant volatility criticism made by Bates (2003), Alexander (2008b), Christoffersen et al. (2012b), we statistically examine the comparative performance among a range of leverage GARCH models. Various asymmetric forms are considered, including exponential, non-linear asymmetric and a GJR-model as well as certain alternative adaptations. Our choice is directed not only by the loss function but also by parsimony. We apply the maximum likelihood estimator (MLE) to determine their parameter estimates, which are converted to their risk-neutral versions in order to generate the imputed option price using a Monte-Carlo (MC) simulation technique. The exception is the Heston and Nandi model because of its analytical solution. The option result is compared with the predicted option price from the Black-Scholes and Gram-Charlier models. Model performance is assessed using an out-of-sample approach, because of its superior robustness (Duan and Zhang 2001).

By running the GARCH models over the historical S&P500 data, we find that the asymmetric GARCH model is advantageous in characterising the index volatility. According to the value of the maximum likelihood function, the standard GARCH model shows the lowest likelihood and the non-linear GARCH model shows the greatest likelihood value when describing the data. For the S&P500 option valuation results, our finding is consistent with Hardle and Hafner (2000), Christoffersen and Jacobs (2004b), who favour the non-linear asymmetric GARCH model than other models, given its least overall option valuation errors. However, the standard GARCH model shows the least valuation results in pricing the S&P500 call options. Though this result is not in line with majority findings in literature, it is consistent with Chiang and Huang (2011) which support the simple form GARCH model.

Although various GARCH specifications are developed to date either in its model complexity and advances, a comparative analysis among these parsimonious GARCH models for option pricing remains largely unexplored in the literature. Our main contribution is to fill this gap by evaluating whether these simpler GARCH models provide convenience but also sufficient precision in option pricing. To our consideration, a practical model should be as simple as possible, but not simpler than a form adequately capturing volatilities' essential stylised facts, such as clustering, time-variation, and leverage effect. By revising these models to their parsimonious formulations, our amended GARCH model parameters can be easily estimated under commonly used statistical packages such as R, Eviews, S-plus and Stata, which is a practical advantage over other self-owned models proposed in literature.

The reminder of this chapter is organised as follows. Section 3.2 presents our methodology, which includes GARCH model estimation procedure and the numerical methods for valuing options. Section 3.3 describes the data and discusses the empirical results. Section 3.4 discusses the volatility modelling as well as S&P 500 option valuation results. Section 3.5 concludes.

## 3.2 Models and evaluating procedure

### 3.2.1 The GARCH models

In this section we introduce methodology including models and the valuation procedures applied. Table 3.1 presents the selected GARCH(1,1) models used in volatility modelling and option valuation[2]. The most fundamental is the GARCH model of Bollerslev (1986) with ordinary formulation. Afterwards it is exponential GARCH model (E-GARCH) proposed by Nelson (1991), in which volatility evolution is measured under a logarithmic form to ensure their positive

**Table 3.1 GARCH (1,1) models under physical measure $\mathbb{P}$**

| | |
|---|---|
| Basic GARCH of Bollerslev (1986)<br>$\ln(S_t / S_{t-1}) = r_f + \varepsilon_t$<br>$h_t = \omega + \beta h_{t-1} + \alpha \varepsilon_{t-1}^2$ | (3.1) |
| E-GARCH of Nelson (1991)<br>$\ln(S_t / S_{t-1}) = r_f + \varepsilon_t$<br>$\ln(h_t) = \omega + \beta \ln(h_{t-1}) + \alpha(\varepsilon_{t-1} / \sqrt{h_{t-1}}) + \gamma(\lvert \varepsilon_{t-1} \rvert / \sqrt{h_{t-1}} - \sqrt{2/\pi})$ | (3.2) |
| GJR-GARCH of Glosten et al. (1993)<br>$\ln(S_t / S_{t-1}) = r_f + \varepsilon_t$<br>$h_t = \omega + \beta h_{t-1} + \gamma I_{\varepsilon_{t-1}<0} \varepsilon_{t-1}^2 + \alpha \varepsilon_{t-1}^2$ | (3.3) |
| NA-GARCH of Engle and Ng (1993)<br>$\ln(S_t / S_{t-1}) = r_f + \varepsilon_t$<br>$h_t = \omega + \beta h_{t-1} + \alpha h_{t-1}(\varepsilon_{t-1} / \sqrt{h_{t-1}} - \gamma)^2$ | (3.4) |
| HN-GARCH model of Heston and Nandi (2000)<br>$\ln(S_t / S_{t-1}) = r_f + \lambda h_t + \varepsilon_t$<br>$h_t = \omega + \beta h_{t-1} + \alpha(\varepsilon_{t-1} / \sqrt{h_{t-1}} - \gamma \sqrt{h_{t-1}})^2$ | (3.5) |
| For all five GARCH specifications, $S_t$ denotes the underlying price at time *t*. $h_t$ denotes the conditional variance at time *t*, $\varepsilon_{t-1}$denotes the unexpected return at time *t-1* with distribution $N(0,h_t)$ given information set $F_{t-1}$. $\omega$ is model intercept, $\beta_i$ describes the volatility persistence, $\alpha_i$ characterises the effect of information shock, $\gamma_1$ is the asymmetric parameters describing the leverage effect. $I_{\varepsilon(t-1)<0}$ is the indicator function having a value of 1 when $\varepsilon_{t-1}<0$. To ensure stationarity, all GARCH models except H-N GARCH are defined to have the mean equation that $\ln(S_t/S_{t-1}) = r_f + \varepsilon_t$, where $r_f$ is the risk-free rate. | |

[2] The GARCH models estimation in our study are implemented using the rugarch and fOptions packages in R, with contributions given by Ghalanos (2016) and Wuertz et al. (2015).

values. The GJR-GARCH model is proposed in Glosten et al. (1993) and applies the indicator function to reflect the leverage effect. The last two models are non-linear asymmetric GARCH models including the NA-GARCH model of Engle and Ng (1993) and the HN-GARCH of Heston and Nandi (2000). The later form is in affined structure which contains an analytical solution for the option pricing.

Besides the Basic GARCH form, the asymmetric variants have the capacity to capture the leverage effect. It is recognised that negative return in stock market raise volatility more than the same magnitude positive return does (Black 1976; Hentschel 1995; Heston and Nandi 2000). This is because the decrease of stock price reduce its equity value relative to debt value, as a result the riskiness is increased accordingly which results in higher volatility (Black 1976).

**Table 3.2 The news impact function**

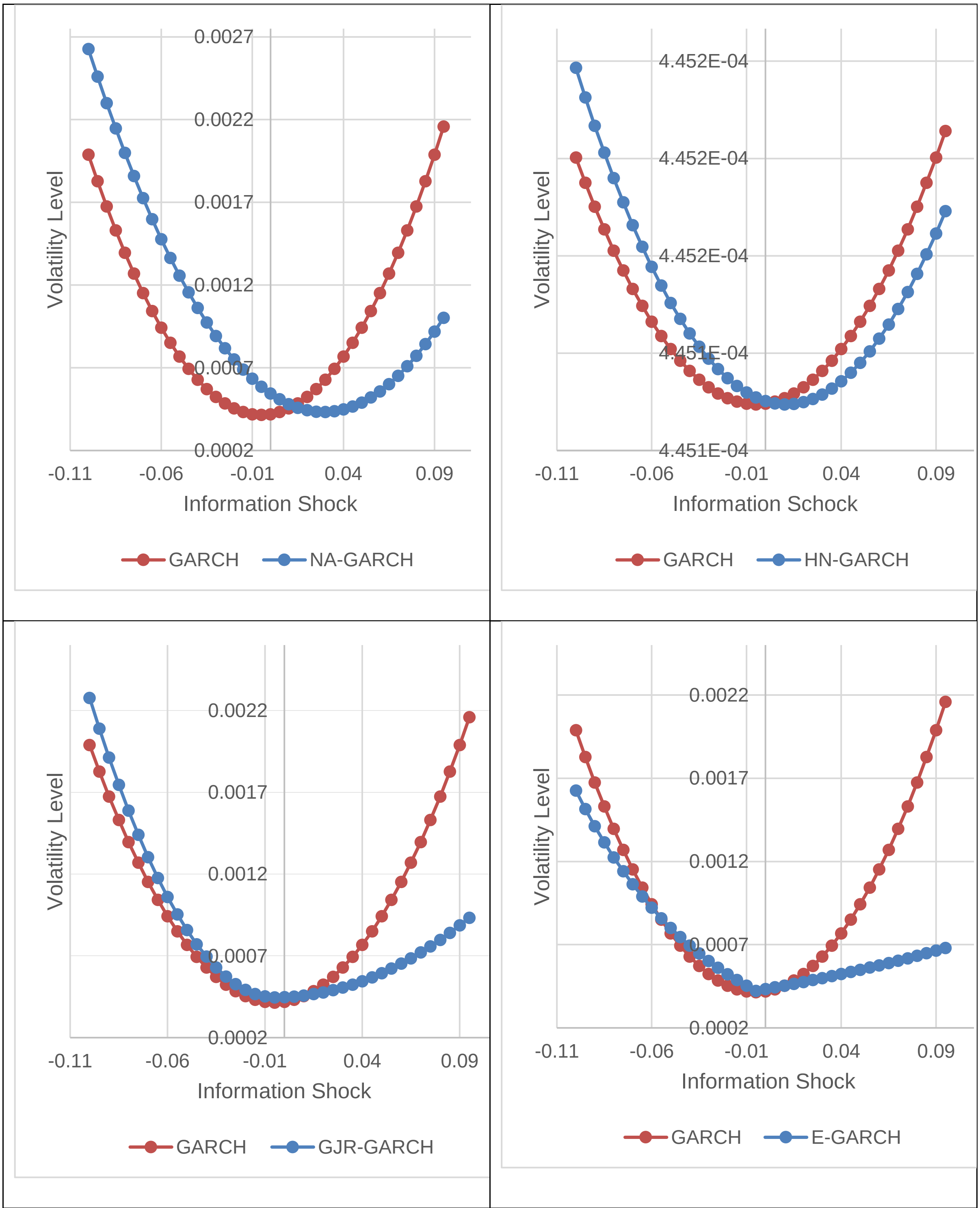
| |
|---|
| The NIF for the E-GARCH |
| $\text{when } \varepsilon_{t-1} > 0,\ h_t = \sigma^{2\beta} \exp(\omega - \gamma\sqrt{2/\pi}) \exp[(\gamma + \alpha)/\sigma * \varepsilon_{t-1}]$ <br> $\text{when } \varepsilon_{t-1} < 0,\ h_t = \sigma^{2\beta} \exp(\omega - \gamma\sqrt{2/\pi}) \exp[(\gamma - \alpha)/\sigma * \varepsilon_{t-1}]$ |
| The NIF for the GJR-GARCH |
| $\text{when } \mathrm{v}_{t-1} > 0,\ \mathrm{h}_t = \omega + \beta\sigma^2 + \alpha\sigma^2(1-\gamma)^2 v_{t-1}^2$ <br> $\text{when } \mathrm{v}_{t-1} < 0,\ \mathrm{h}_t = \omega + \beta\sigma^2 + \alpha\sigma^2(1+\gamma)^2 v_{t-1}^2$ |
| The NIF for the NA-GARCH |
| $\text{when } \varepsilon_{t-1} > 0,\ h_t = \omega + \sigma^2(\alpha(\varepsilon_{t-1}/\sigma - \gamma)^2 + \beta)$ <br> $\text{when } \varepsilon_{t-1} < 0,\ h_t = \omega + \sigma^2(\alpha(\varepsilon_{t-1}/\sigma + \gamma)^2 + \beta)$ |
| The NIF for the HN-GARCH |
| $\text{when } \varepsilon_{t-1} > 0,\ h_t = \omega + \beta\sigma^2 + \alpha(\lvert \varepsilon_{t-1} \rvert/\sigma - \gamma\sigma)^2$ <br> $\text{when } \varepsilon_{t-1} < 0,\ h_t = \omega + \beta\sigma^2 + \alpha(\lvert \varepsilon_{t-1} \rvert/\sigma + \gamma\sigma)^2$ |
| Notes: For each news impact function (NIF) of the their respective GARCH model, two equations are presented in which one equation describe the volatility behaviour when unexpected return arise from good news ($\varepsilon_t$>0) and another equation describes the volatility behaviour arise from bad news ($\varepsilon_t$<0). $\sigma$ is the unconditional variance of S&P500 price returns, $\alpha,\ \beta,\ \gamma$ are model parameters reported in Table 3.1. $v_t$ is identical .independent distributed residuals in that $v_t \sim N(0,1)$. |

In order to distinguish how each asymmetric model responds to the new arrival information, the news impact function of each model is derived and summarised in Table 3.2. In line with Engle and Ng (1993), through the conditional volatility equation in equations (3.1) to (3.5) we could have the unexpected return `ε' which defines the unexpected increase or decrease in the price of underlying asset. Hence an unexpected increase denoted by a positive `+ε' suggest the arrival of good news whereas an unexpected decrease denoted by a negative `-ε' suggest the arrival of bad news. As consequence the arrival of good and bad news could result in different information shocks to the volatility. A large absolute value of ε indicates the news is `significant', which results in a considerable change in the volatility and asset price.

Figure 3.1 compares the NIC (news impact curve) for the various GARCH models relative to the basic form. This reveals that the leverage effect causes the NIC to shift laterally for the NA-GARCH and HN-GARCH models, with a positive (negative) unexpected return shifting the NIC to the right (left), but to rotate for the GJR-GARCH and E-GARCH models. Notice that the E-GARCH model flattens the NIC and the negative shocks increase the volatility less than for the GJR-GARCH. Overall, the NIC show that the negative shocks always increase volatility more than the positive shocks, which is consistent with Engle and Ng (1993), Ding et al. (1993), Hentschel (1995), and Christoffersen and Jacobs (2004b).

Two approaches are normally used in the estimation of the GARCH style models. The first is based on the maximum likelihood principle, see for example Amin and Ng (1993), Bollerslev and Mikkelsen (1996), Hardle and Hafner (2000), Heston and Nandi (2000), Christoffersen and Jacobs (2004b), Christoffersen et

## Figure 3.1 The News Impact Curves




Notes. The news impact curve (NIC) for each asymmetric GARCH model (solid line) is superimposed on the NIC from the symmetric GARCH model (dashed line). The vertical axis denotes the conditional variance whereas the horizontal axis denotes the value of the unexpected return $\varepsilon$ that ranges from left (-1) ro right (1).

al. (2010b), while the second on the non-linear squared method (Engle and Mustafa 1992; Christoffersen and Jacobs 2004b; Christoffersen et al. 2012b; Kanniainen et al. 2014). Since the latter is computationally onerous due to the need to minimise the model price market option price squared difference (Broadie and Detemple 2004; Christoffersen and Jacobs 2004b; Duan and Yeh 2011), we prefer to use the MLE because of its computational ease and efficiency.

By MLE, the parameter estimates are determined from maximising the log-likelihood function L (Bollerslev 1986), where,

$$\ln L = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=2}^{T}\ln(h_t) - \sum_{t=2}^{T}(\ln\frac{S_{t+1}}{S_t} - r_f)^2 / 2h_t^2 \qquad (3.6)$$

where $h_t$, $S_t$, $r_f$ respectively denotes the conditional variance, underlying asset price, and risk-free rate[3] at time *t* with maturity *T-t*.

The MLE for (3.6) are obtained using a sample which belongs in the real-world physical measure P, but options are normally valued under the risk-neutral measure Q. In a world of complete market and no-arbitrage, market participants under risk-neutrality are indifferent to risk and all assets are discounted at the risk-free rate. The procedure for translating from P to Q is developed by Duan (1995), Hardle and Hafner (2000), Heston and Nandi (2000), Christoffersen and Jacobs (2004b). It involves using the LRNVR (locally risk-neutral valuation relationship) to represent the future possible evolutions of the underlying asset price so that the option payoff at maturity can be discounted at the risk-free rate.

[3] We assume the risk-free rate to be constant over the period *T-t*.

Let $R_t = S_t / S_{t-1}$, $E^P[R_t | F_{t-1}]$ and $E^Q[R_t | F_{t-1}]$ denote the expected return under the physical probability measure and risk neutral measure, respectively and $F_{t-1}$ be the information set at time t-1, then the following conditions need to be satisfied under $\mathbb{Q}$,

$$\begin{aligned} &E^Q[\exp(\ln S_t / S_{t-1}) | F_{t-1}] = \exp(r_f), \\ &Var^Q[R_t | F_{t-1}] = Var^P[R_t | F_{t-1}] = h_{t-1} \end{aligned} \tag{3.7}$$

which implies that under the risk-neutral measure:

$$E^Q[\ln S_t / S_{t-1}] = E^Q[R_t] = r_f - \frac{1}{2} h_t + v_t^* \sqrt{h_t},\ \mathrm{v}_t^* \sim N(0,1) \tag{3.8}$$

with $v_t^* = \xi_t / \sqrt{h_t} - \frac{1}{2}\sqrt{h_t}$. Then we derive the conditional volatility equations for each GARCH model under measure $\mathbb{Q}$, which are presented in Table 3.3.

We determine the option price from a Monte Carlo simulation based on the risk-neutral GARCH models. Although other numerical methods such as a trinomial-tree and the Edgeworth expansion are available (Ritchken and Trevor 1999; Cakici and Topyan 2000; Duan et al. 2003), their comparative merits lies in valuing American options and not in computational ease since certain parameter configurations can seriously slow down the algorithm (Lyuu and Wu 2004; Hsieh and Ritchken 2005; Chen et al. 2012). In comparison, the Monte Carlo simulation has more feasible advantages in its algorithm configuration and implementation.

**Table 3.3 The GARCH (1,1) models under risk-neutral measure $\mathbb{Q}$**

| | |
|---|---|
| The GARCH model<br>$\ln S_t / S_{t-1} = R_t = r_f - \frac{1}{2}h_t + v_t^* \sqrt{h_t}$<br>$h_t = \omega + \beta h_{t-1} + \alpha h_{t-1}(v_{t-1}^* - 1/2\sqrt{h_{t-1}})^2$ | (3.9) |
| The E-GARCH model<br>$\ln S_t / S_{t-1} = R_t = r_f - \frac{1}{2}h_t + v_t^* \sqrt{h_t}$<br>$\ln h_t = \omega + \alpha_1(v_{t-1}^* - 1/2\sqrt{h_{t-1}}) + \gamma_1(\lvert v_{t-1}^* \rvert - 1/2\sqrt{h_{t-1}} - 2/\pi) + \beta \ln(h_{t-1})$ | (3.10) |
| The GJR-GARCH model<br>$\ln S_t / S_{t-1} = R_t = r_f - \frac{1}{2}h_t + v_t^* \sqrt{h_t}$<br>$h_t = \omega + \beta h_{t-1} + \alpha_1 h_{t-1}(v_{t-1}^* - 1/2\sqrt{h_{t-1}})^2 + \gamma_1 h_{t-1} I_{v_{t-1<0}}(v_{t-1}^* - 1/2\sqrt{h_{t-1}})^2$ | (3.11) |
| The NA-GARCH model<br>$\ln S_t / S_{t-1} = R_t = r_f - \frac{1}{2}h_t + v_t^* \sqrt{h_t}$<br>$h_t = \omega + \beta h_{t-1} + \alpha_1 h_{t-1}(v_{t-1}^* - 1/2\sqrt{h_{t-1}} - \gamma_1)^2$ | (3.12) |
| The HN-GARCH model<br>$\ln S_t / S_{t-1} = R_t = r_f - \frac{1}{2}h_t + v_t^* \sqrt{h_t}$<br>$h_t = \omega + \beta h_{t-1} + \alpha_1(v_{t-1}^* - (0.5 + \lambda + \gamma_1)\sqrt{h_{t-1}})^2$ | (3.13) |
| Note: All five GARCH models derived under the risk neutral measure, $\omega$, $\alpha$, $\beta$, and $\gamma$ are respectively parameters defined in Table 3.1. $v_t$* is i.i.d. residual with $v_t$* ~ N(0,1) but is defined under the risk-neutral measure. | |

To obtain option price using the Monte Carlo simulation method, first of all, the underlying asset price *S* is firstly simulated from time point *0* to *T*,

$$S_T = S_0 \exp\left( Tr_f - \frac{1}{2}\sum_{i=1}^{T} h_i + \sum_{i=1}^{T} \xi_i \right). \qquad (3.14)$$

Then with terminal asset price $S_T$ and pre-specified exercise price K, the simulated call option price is,

$$C_T^{GH}(n) = \exp\left(-Tr_f\right) * \frac{1}{n}\sum_{i=1}^{n}\max\left(S_{T,i} - K, 0\right) \tag{3.15}$$

where $i$ denotes the $i$th order simulation and $n$ is the number of simulations. Following this step, to improve the accuracy of the simulation, the variance reduction methods of antithetical variation procedure of Boyle et al. (1989) and the control variate procedure used in Schmitt (1996) are employed during the computation. To implement the antithetical variation, two random variables sequence $\varepsilon_1$ and $-(\varepsilon_2)$ are generated with relationship $\varepsilon_1+\varepsilon_2= \varepsilon$. Therefore $\varepsilon_1$ and $\varepsilon_2$ are from the same path but have opposite signs. Then we take the average values of the simulated option price. The intuition here is that in case the simulation from one ε path overestimates the terminate value, then another simulation from the opposite path will have the underestimation. As a result the bias can be offset and therefore improve the accuracy. While for the control variation technique, the option price $C^{gBm}$ is computed under the geometric Brownian motion to have the control variable because of its closed form solution, i.e. the Black-Scholes formula, for the $C^{BS}$ exists,

$$\begin{aligned} C^{gBm} &= \frac{1}{n}\exp\left(-r*T\right)\max\left(S_{T,i} - K, 0\right) \\ S_{T,i} &= S_0\exp\left(\left(r - \frac{1}{2}h_0\right)T + \sqrt{h_0 T} * v_i\right) \end{aligned} \tag{3.16}$$

where $h_0$ is the initial variance which is approximated by the sample unconditional variance. Subsequently, the resulting adjusted GARCH option price $C^{GH}$ is calculated as,

$$
\begin{aligned}
C^{GH} &= C^{GH} - q(C^{gBm} - C^{BS}) \\
q &= \frac{Cov\left(C^{GH}, C^{gBm}\right)}{Var\left(C^{gBm}\right)}
\end{aligned} \tag{3.17}
$$

As is customary, the adjusted estimated GARCH price $C^{GH}$ refers to the GARCH option price.

### 2.2.2 Benchmark Models

We employ two continuous time models to assess each GARCH model's option valuation performance. They are the Black-Scholes (BS) model and the Gram-Charlier model of Backus et al. (2004). Both models are continuous time model with the closed form solution, but the latter has more accurate distribution measurement on skewness and kurtosis. According to Christoffersen (2012), equity market exhibits negative skewness because large negative returns occurs more often than the large positive returns, and positive kurtosis because large returns (either positive or negative) occur more likely than predicted from a normal distribution. We expect these two benchmarks to cover most eventualities and by so doing have a more fully-fledged assessment of the comparative performance of the various GARCH models.

To express the models, let *S, K, σ, r, T* respectively denotes the underlying asset price, exercise price, volatility, risk-free rate, and option maturity. Let N(·) be the cumulative normal density function, and $c^{BS}$ and $p^{BS}$ be the value of call option and put option respectively, then we have the Black-Scholes model with following expression,

$$
\begin{aligned}
c^{BS} &= SN(d_1) - K\exp(-rT)N\left(d_2\right) \\
p^{BS} &= K\exp(-rT)N\left(-d_2\right) - SN\left(-d_1\right) \\
d_1 &= \left[\ln(S/K) + (r+\sigma^2/2)T\right] \Big/ \left(\sigma\sqrt{T}\right) \\
d_2 &= d_1 - \sigma\sqrt{T}.
\end{aligned} \tag{3.18}
$$

The Gram-Charlier model extends the Black-Scholes model by allowing for the skewness and kurtosis of underlying asset distribution. Based on the feature of positive kurtosis, it seems that the Gram-Charlier model is analogous to the discrete time GARCH model with fat-tail distribution, and therefore a natural benchmark after the Black Scholes model. By letting $\zeta_3$ and $\zeta_4$ be the parameters measuring the skewness and kurtosis of the underlying return distribution, respectively, the Gram-Charlier option price $C_{GC}$ can be written as:

$$
\begin{aligned}
C_{GC} &= S_t N\left(d_1\right) - K\exp\left(-rT\right) N\left(d_1 - \sqrt{T}\sigma\right) \\
&\quad + S_t\phi(d_1)\sigma\left[\frac{\zeta_3}{3!}\left(2\sqrt{T}\sigma - d_1\right) - \frac{\zeta_4}{4!\sqrt{T}}\left(1 - d_1^{\,2} + 3d_1\sqrt{T}\sigma - 3T\sigma^2\right)\right] \\
\zeta_3 &= E\left(R_{t+1} - \mu + \frac{1}{2}\sigma^2\right)^3 / \sigma^3\sqrt{T} \\
\zeta_4 &= E\left(R_{t+1} - \mu + \frac{1}{2}\sigma^2\right)^4 / \sigma^4 T - 3T
\end{aligned} \tag{3.19}
$$

where $\phi(\cdot)$ is the standard normal density, $\mu$ and $R$ are respectively the mean and daily returns underlying asset price. The put option price is obtained through the put-call parity:

$$
P_{GC} = C_{GC} + K\exp(-rT) - S_t\,.
$$

By setting the third and fourth moment equal to zero the Gram-Charlier model simplifies to Black-Scholes formula (3.18).

### 3.3. Data and Preliminary analysis

Historical daily data on the stock index S&P500 (Standard & Poor 500) is used for model estimation, and the CBOE (Chicago Board Options Exchange) traded stock index option is used to examine the deviations of model price to the market option price. The S&P500 index, with a constituency made up of the 500 largest publicly traded US companies, is one of the most closely followed security indices and generally considered to be the best representative of US stock market behaviour. The underlying S&P500 index option is traded on CBOE, which is the second most active options market in the U.S. with the largest open interest (Rubinstein 1994). These options are European-style, do not have an early exercise feature, which can complicate the valuation procedure. Also, unlike commodity options that tend to experience jump in price movement, the S&P 500 index option market provides the best platform for conducting option valuation and comparing valuation performance with many other studies such as Rubinstein (1994), Bakshi et al. (1997), Dumas et al. (1998), Heston and Nandi (2000), Christoffersen and Jacobs (2004b), and Christoffersen et al. (2008).

The original data series span from June 28, 1996 to June 28, 2016 with 5218 observations. The mid-value between daily high and low price of daily quote are used to best approximate the S&P500 daily price movement. Prior to the model estimation, we examine the potential structural breaks over the data series in order to best avoid any necessary breaks during the model estimation. Although a lengthy time series enable us to obtain precise model estimates, significant structural breaks could cause inaccurate model estimation also. We attempt to identify any potential breaks over the data and in the meantime avoid significant breaks to ensure a sufficient time span for our models estimation(Heston and Nandi 2000; Christoffersen 2012).

In determining approaches for identifying the potential structural breaks, existent statistical methods could be broadly classified into two categories, the fluctuation tests and tests based on F-statistics. For the former test its principal is that for any given process one assess its cumulative sums or moving sums of residuals or parameter estimates, which under the null hypothesis is evolved with a functional central limit and with limited fluctuation. The alternative hypothesis under assessment is the presence of structural change, which raises the increased fluctuation. For the tests based upon F-statistics, these are designed for examining the single shift in time series structural with unknown timing, which calculate the chow statistics for each conceivable points of structural change with a certain interval. The test rejects its null hypothesis in case its statistics value exceeds the given critical value.

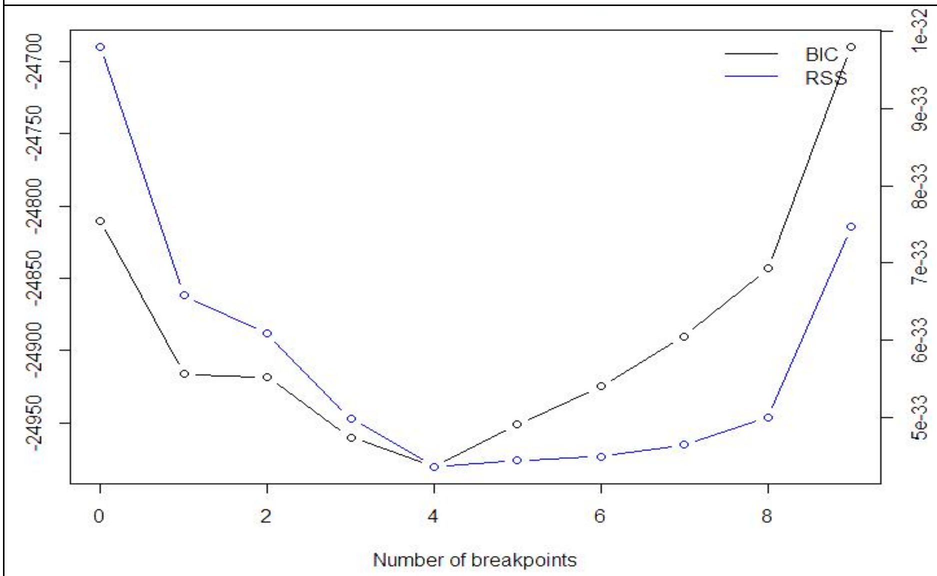
For our case of identifying multiple breaks over times series in potential, we apply the general methodology of Bai and Perron (2003)[4] which is suggested as a formal approach in estimating breaks (Kleiber and Zeileis 2008). To examine the potential structural break, we first establish the following regression model over the S&P500 index with monthly frequency between January 1986 to June 2016,

$$R_t = b_1 R_{t-1} + b_{12} R_{t-12} + e_t \tag{3.20}$$

[4] The Bai and Perron (2003) multiple structural break test is available in R's strucchange package.

$R_t$ , $R_{t-1}$, and $R_{t-12}$ denote the monthly logarithmic return at time t, t-1, and t-12, respectively. $b_1$ and $b_{12}$ are coefficients for regressors, $\varepsilon_t$ is the ordinary least square (OLS) residual at time t. Monthly S&P500 index return series is used for the model estimation[5]. We use the explanatory variables with one and twelve lag length to examine if coefficients on any explanatory variables are constant or vary over time. We assume that m breakpoints, and m+1 segments exist over the data, and that each data segment has a different coefficient regression relationship.

**Figure 3.2 BIC and RSS breakpoint testing results**




Notes: BIC and RSS results for the assumed m breakpoints, their BIC and residual sum of squares (RSS) values are estimated, by minimising the above two values of each breakpoint. Clearly the 4-breakpoint setting best describes the data.

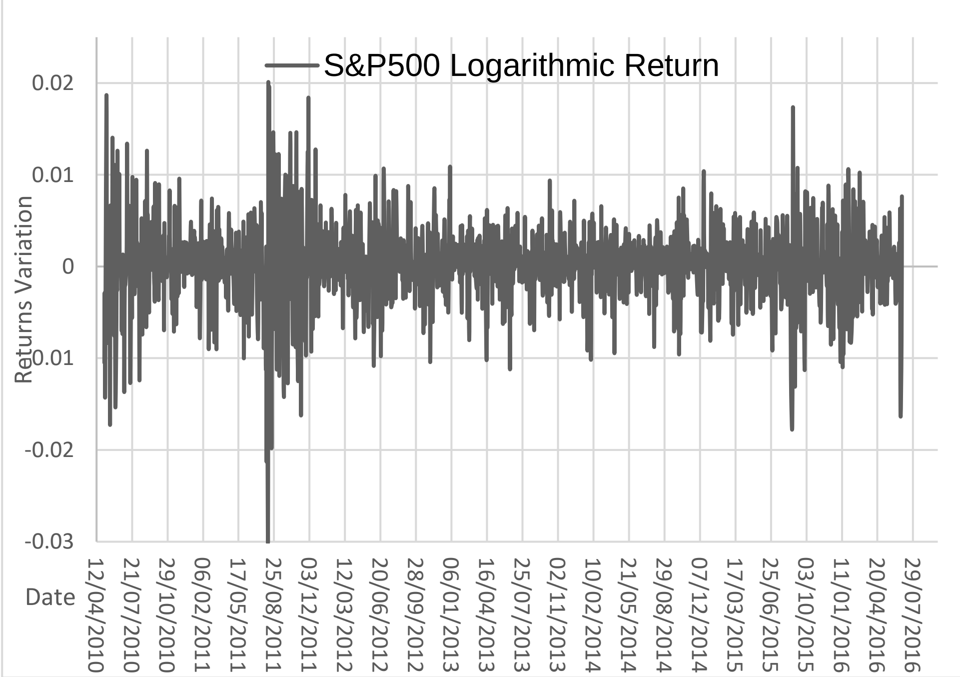
Figure 3.2 reports the number of break points identified by minimizing the residual sum of squares (RSS) and the Bayesian information criterion (BIC).

[5] We also examined the structural breaks using the daily price series with the results indicating similar period of potential structural breaks. However, the daily series is not advisable since computation becomes burdensome as data frequency increase.

Structural break points from February.1999, March.2004, October 2008, and April.2010 are identified[6]. Thus, we confine our sample from May 3, 2010 to June 28, 2016 for the model estimation use, which contains a total of 1607 observations. Thus we confine our sample from May 3, 2010 to June 28, 2016 for the model estimation use, which contains a total of 1607 observations.

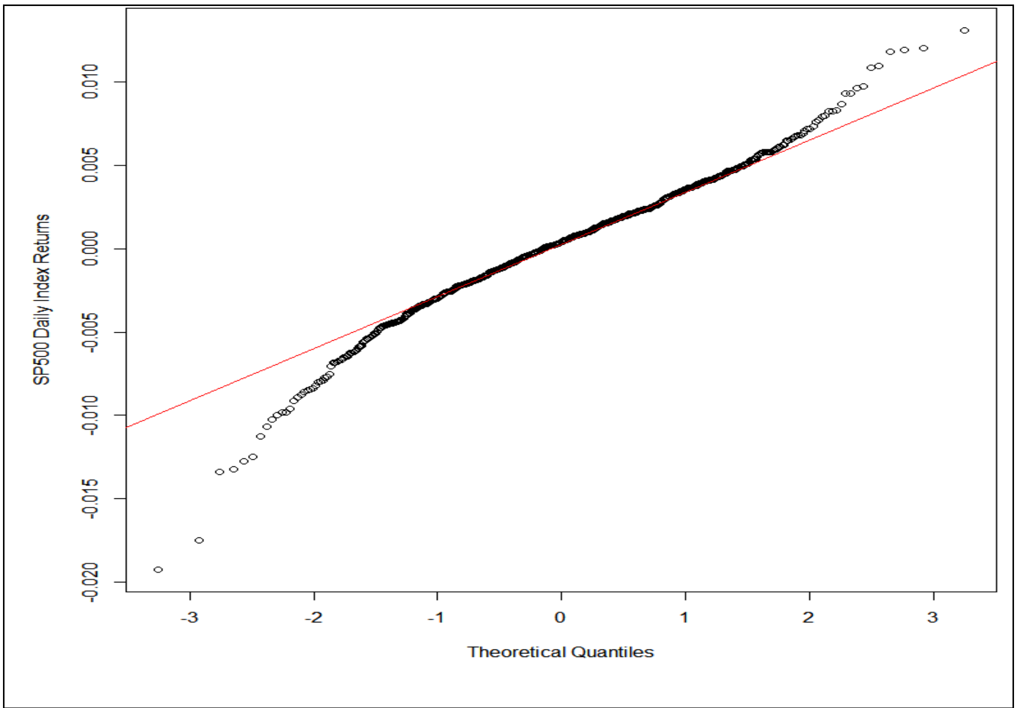
Figure 3.3 plots the confined data series of S&P500 index returns spanning from May 3, 2010 to June 28, 2016, indicating the observable time variation of returns' volatility. The normality of the S&P500 returns is assessed from the quantile-quantile plot of the confined time span, presented in Figure 3.4. The deviation is pronounced on both sides of the 45-degree line, suggesting a non-normal distribution with fat tails. The Jarque-Bera test value of 234.03 further confirm

**Figure 3.3 S&P500 Index Plot (May 3, 2010 to June 28, 2016)**




[6] Significant market events related to these break points include the Gulf War in February 1991, the starting collapse of the 1999 tech bubble, the launch of VIX (volatility index) in March 2004, the European debt crisis in the end of 2008, and the explosion of the British Petroleum oil rig in Gulf of Mexico in April 2010.

**Figure 3.4 The quantile-quantile plot**




Note: The quantile-quantile plot of the S&P500 daily returns from 03.May.2010 to 28.June.2016. The deviation of the plot over the left and right side of the 45 degree line suggest a fat tails distribution of our data.

this result, with a skewness value of -0.413 indicating a left skewness and a Kurtosis value of 4.336 suggesting a stronger peak and heavier distribution tails.

The estimation process also requires information on the daily risk-free rate and dividend rate. We use the continuously compounded treasury bill (T-bill) rate due to its short-maturity which has a close match to the S&P500 options maturity (Bakshi et al. 1997; Hardle and Hafner 2000; Christoffersen and Jacobs 2004b). Possible timing mismatches between the T-bill and the option, are interpolated from:

$$r_{f,T} = \frac{r_{f,T_2} - r_{f,T_1}}{T_2 - T_1} * (T - T_1) + r_{f,T_1} \tag{3.21}$$

where $T_1 < T < T_2$.

Regarding the dividend yield on the S&P500 index, our models do not accommodate the dividend yield for the sake of simplicity. Thus it is important to have a dividend excluded return series for model estimation. Assuming 360 working days a year, we can also have the present value of dividend $d_{pv}$, where

$$d_{pv} = \frac{d}{(1+r_f)^{T/360}} \tag{3.23}$$

*d* is value from S&P500 daily dividend index, *T* is the time to maturity of option. We subtract $d_{pv}$ from daily return of S&P500 index to have the dividends excluded from the S&P500 return series for the GARCH model parameters estimation.

Data on the S&P 500 index option was collected from July 03, 2014 to June 16, 2016, although several necessary exclusions are made. Options with a maturity less than six days are excluded because the premiums of short-term maturities encapsulate limited volatility information, while those with a maturity more than 100 days are excluded because of illiquidity distortions[7]. Furthermore, short- and long- term maturity option contracts are highly sensitive to the nonsynchronous option values and other transaction-related measurement errors (Bakshi et al. 1997; Dumas et al. 1998). Options are also excluded on their degree of moneyness. Deep in-the-money and out-of-the-money contracts are excluded because of having small time premiums and little embodied volatility information. These contracts that are not actively traded, tend to be illiquid, and Duan and Zhang (2001) report that the implied volatility information

[7] Previous studies with similar data filtration can also be found in Bakshi et al. 1997. However, unlike the data filtration applied in Bakshi et al (1997), Christoffersen and Jacob (2004), and Christoffersen et al (2012), we exclude the option data with maturity more than 100 days is because that the trading of these option contracts are illiquid and result in inconsistent historical data over time.

is distorted and unreliable. Finally, we adopt the practice of Dumas et al. (1998), Heston and Nandi (2000), Duan and Zhang (2001), Christoffersen and Jacobs (2004b), Christoffersen et al. (2008), Christoffersen et al. (2012a), and use Wednesday option data only because very few holidays occur on this weekday so one could have sufficiently long and continuous time series.

### 3.4 Volatility modelling and S&P 500 option valuation results

#### 3.4.1 Model estimating results

The estimated GARCH model parameters under the physical measure $\mathbb{P}$ are reported in Table 3.4. As parameters defined in Table 3.1, for each GARCH model, $\omega_1$, $\alpha_1$, $\beta_1$, are respectively the constant, the unexpected return coefficient, the volatility clustering coefficient. $\lambda$ is the unit risk premium parameter and $\gamma_1$ is asymmetry parameter capturing the leverage effect. The likelihood Ratio (LR) test with simple GARCH and NA-GARCH used respectively as criteria.

Among the estimated parameters, the *βs* show the largest weight amongst the parameters, which suggests that volatility clustering is statistically a most pronounced fact. The intercept $\omega_1$ which represents average unconditional variance, have values about zero for all models except for E-GARCH. The $\gamma_1$s have positive values and are statistically significant for all asymmetric GARCH

models, which indicate a significant leverage effect in the S&P500 index. The estimated $\lambda$ shows the value of -2.537, implying the negative risk premium amounts to 2.537 of the S&P500 index returns. Overall, the sign and size of

**Table 3.4 GARCH models estimating results under measure $\mathbb{P}$**

| | $\omega_1$ | $\alpha_1$ | $\beta_1$ | $\gamma_1$ | $\lambda$ | Log-Likelihood | LR test GARCH | LR test NA-GARCH |
|---|---|---|---|---|---|---|---|---|
| GARCH | 4E-06 | 0.136 | 0.825 | | | 5340.47 | Retain $H_0$ | Retain $H_1$ |
| | (1.95) | (3.06) | (13.879) | | | | (0.00) | (142.04) |
| E-GARCH | -0.445 | -0.243 | 0.952 | 0.117 | | 5405.72 | Retain $H_1$ | Retain $H_1$ |
| | (-4.88) | (-6.38) | (96.513) | (4.28) | | | (130.5) | (11.54) |
| GJR-GARCH | 4e-6 | 0 | 0.912 | 0.239 | | 5390.42 | Retain $H_1$ | Retain $H_1$ |
| | (0.74) | (8E-6) | (2.27e+3) | (17.49) | | | (99.9) | (42.14) |
| NA-GARCH | 3E-06 | 6.027e-2 | 0.704 | 1.829 | | 5411.49 | Retain $H_1$ | Retain $H_0$ |
| | (0.37) | (72.14) | (322.79) | (346.69) | | | (141.9) | (0.00) |
| HN-GARCH | 2.67E-82 | 1.82E-6 | 6.67E-01 | 4.20E+02 | -2.537 | 5408.92 | Retain $H_1$ | Retain $H_1$ |
| | (0.35) | (5.14) | (43.69) | (11.18) | (2.01) | | (135.9) | (5.14) |

Maximum Likelihood Estimates of GARCH (1,1) specifications using S&P500 index log-return from 3.May.2010 to 28.June.2016 with 1607 observations. t-values for each parameter is reported in parentheses.. $\omega_1$, $\alpha_1$, $\beta_1$, respectively refer to the constant parameter, information shock parameter, and persistence parameter. $\lambda$ in the HN-GARCH model is the unit risk premium . $\gamma_1$ is asymmetry parameter examining the leverage effect. The LR (Likelihood ratio) tests the null the hypothesis $H_0$ that a particular model has the data fit better than the restricted model, which are the models depicted in parenthesis, against the alternative hypothesis $H_1$ that the alternative model fits the data better.

estimated parameters lies with expectation[8], and are in line with previous findings such as Heston and Nandi (2000), Christoffersen and Jacobs (2004b), and Hsieh and Ritchken (2005).

Finally, the maximum likelihood estimation results indicate that the NA-GARCH model fits the data best, followed by HN-GARCH and then E-GARCH. The LR (Likelihood ratio) test endorses this result. When using the Basic GARCH model as a criterion, the test results indicate all leverage GARCH models outperform the Basic GARCH model in data fitting. When the NA-GARCH model is used as criterion, the test results suggest that the remaining asymmetric models are all less adequate.

### 3.4.2 The option pricing procedure and valuation results

We simulate[9] the model option price for each of the specified GARCH models. 50,000 repetitions[10] are carried out for our Monte Carlo simulation and the unconditional variance is used as the initial conditional volatility. For the Black-Scholes and Gram-Charlier model, the recent one month standard deviation is used in computing their model option price. Also for the Gram-Charlier model we use the skewness and kurtosis parameters estimated from the historical underlying price for the distribution calibration.

We categorise the option data with the following criteria. The various option contracts investigated are categorised by their degree of moneyness and by maturity. Moneyness M is characterised by the ratio of strike price to spot price.

[8] We also estimated all GARCH models over the historical 20 years S&P 500 index returns, the resulting estimates are very similar.
[9] Notice that the Monte Carlo simulation apply to all our GARCH models but not necessarily for the H-N GARCH due to it's closed-form solution in option pricing.

[10] In original study of Duan (1995), 5000 times simulation is used to obtain reasonable option price. In our study we take 50000 times simulation to have assurance of simulation accuracy. The similar treatment can also be found in Christoffersen et al. (2012).

An option is in-the-money (ITM) if M<0.98, near-the-money (NTM) if $0.98 \le M < 1.02$, out-of-the-money (OTM) if $M \ge 1.02$[11]. In terms of date to expirations for option contracts, an option is short-term (ST) if time to maturity is less than one-month, mid-term (MT) if more than one-month but less than two-months, and long-term (LT) if longer than two-months but shorter than 100 days. Maturity length for each option is identified within parentheses.

Table 3.5a and 3.5b report the valuation results for S&P500 call and put options, respectively. For both tables, the first row shows the model specifications, and the first column indicates the option contract categories according to moneyness and maturity (in parentheses). For each category specified by the moneyness, the valuation errors are measured as the raw value of average relative valuation errors using the loss function of $1/n\sum_{t=1}^{n}\left(C_{i,t}-C_{i,t}^{M}\right)/C_{i,t}^{M}$ to examine the over- and under-pricing of model performance; While for categories labelled by %MSE (percentage mean squared errors), the valuation errors are measured as $1/n\sum_{t=1}^{n}\left[\left(C_{i,t}-C_{i,t}^{M}\right)/C_{i,t}^{M}\right]^{2}$ where $C_{i,t}$ and $C_{i,t}^{M}$ are the *i*th theoretical option price and market option price at time *t*.

For the S&P500 call pricing results, as Table 3.5a shows, in short maturity the E-GARCH model shows the least valuation error of 0.08%, then comes to the NA-GARCH model with slightly higher over-pricing error of 0.09%. Being more specific, the basic GARCH model surprisingly performs best . However, its

[11] For previous studies of Bakshi et al (1997) and Christoffersen and Jacob (2004) their moneyness is defined as the spot price strike price ratio. Though the expression differs but we retain the same indication. Also unlike the moneyness definition given in Bakshi et al (1997) and Christoffersen and Jacob (2004), according to the moneyness definition we narrow down the range of the measurement, i.e.[0.98, 1.02] in our case ,for the at-the-money option data in order to more precise findings.

valuation error increases significantly in the valuation of the out-of-the-money options. For the medium-term category,  the basic GARCH model dominates and the NA-GARCH model also has almost equivalent performance with %MSE of 1.60%. Noticeably the Gram-Charlier model has the least price bias for the medium-term in-the-money contract. For the Black-Scholes model, which adopts the instantaneously constant volatilities in our study, unfortunately has the largest valuation errors for this maturity category. Lastly, for options with long term to expirations, the Gram-Charlier model produces the least pricing bias, with a %MSE of 2.21%, followed by the Basic GARCH model with %MSE of 2.61% valuation error.

Table 3.5b reports the valuation results for the S&P500 put options, in which the NA-GARCH dominates other models in pricing the short term contract. The Basic GARCH model outperforms the remaining models in pricing the medium-term contract, and the Gram-Charlier model shows the least valuation errors in pricing the long term contract. The HN-GARCH model also has good performance in pricing the in-the-money contract and the Gram-Charlier model outperforms other models in pricing the out-of-the money contract across maturities.

## Table 3.5a. S&P500 call valuation errors

| Moneyness (Maturity) | GARCH (3.9) | E-GARCH (3.10) | GJR-GARCH (3.11) | NA-GARCH (3.12) | HN-GARCH (3.13) | Black-Scholes (3.18) | Gram-Charlier (3.19) |
|---|---|---|---|---|---|---|---|
| ITM (ST) | 0.43% | 1.14% | 0.23% | 0.92% | 3.95% | 13.02% | 0.79% |
| NTM (ST) | -4.59% | -0.58% | -14.66% | -3.08% | 44.67% | 36.53% | 11.23% |
| OTM (ST) | 31.87% | -1.11% | -34.90% | -4.17% | 20.33% | 21.31% | 21.49% |
| %MSE (ST) | 0.39% | 0.08% | 1.03% | 0.09% | 18.37% | 16.16% | 1.51% |
| ITM (MT) | -3.40% | 3.38% | -7.34% | -0.59% | 8.23% | 12.89% | -0.10% |
| NTM (MT) | -5.96% | 8.74% | -30.27% | -4.00% | 31.63% | 66.01% | 13.91% |
| OTM (MT) | 6.61% | 15.66% | -43.62% | -8.60% | 70.72% | 18.38% | 18.63% |
| %MSE (MT) | 1.58% | 2.54% | 5.22% | 1.60% | 9.34% | 48.81% | 6.02% |
| ITM (LT) | 0.52% | 11.24% | -9.85% | 3.60% | 14.59% | 2.03% | -1.48% |
| NTM (LT) | 13.15% | 35.60% | -29.52% | 13.75% | 38.29% | 31.01% | 10.82% |
| OTM (LT) | 1.11% | 23.65% | -51.41% | -6.35% | 40.31% | 77.33% | 30.74% |
| %MSE (LT) | 2.61% | 8.38% | 6.25% | 3.76% | 15.47% | 11.83% | 2.21% |

*Note:* ITM, NTM, and OTM are contractions for the valuation results of in-the-money, near-the-money, and out-of-the-money call options, respectively. ST-, MT-, and LT- are abbreviations for option maturities of short term, medium term, and long term to expiration, respectively. For each category specified by the moneyness, the valuation errors are measured as the raw value of average relative valuation errors using the loss function of ${}^{1}/{}_{n}\sum_{t=1}^{n}(C_{i,t} - C_{i,t}^{M})/C_{i,t}^{M}$ to examine the over- and under-pricing of model performance; While for categories labelled by %MSE (percentage mean squared errors), the valuation errors are measured as ${}^{1}/{}_{n}\sum_{t=1}^{n}[(C_{i,t} - C_{i,t}^{M})/C_{i,t}^{M}]^2$ where $C_{i,t}$ and $C_{i,t}^{M}$ are the *i*th model price and market option price at time t.

**Table 3.5b. S&P500 put valuation errors**

| Moneyness (Maturity) | GARCH (3.9) | E-GARCH (3.10) | GJR-GARCH (3.11) | NA-GARCH (3.12) | HN-GARCH (3.13) | Black-Scholes (3.18) | Gram-Charlier (3.19) |
|---|---|---|---|---|---|---|---|
| ITM (ST) | -4.19% | -5.64% | -5.80% | -5.48% | 2.48% | -59.74% | 6.16% |
| NTM (ST) | -9.55% | -13.07% | -18.70% | -12.02% | 40.51% | -19.66% | 14.73% |
| OTM (ST) | -46.93% | -36.62% | -59.20% | -35.68% | 76.59% | -38.77% | 7.20% |
| %MSE (ST) | 7.24% | 5.88% | 14.23% | 4.80% | 32.36% | 24.29% | 7.95% |
| ITM (MT) | -5.47% | -9.32% | -8.67% | -8.14% | 1.55% | -17.16% | 14.37% |
| NTM (MT) | -12.48% | -9.31% | -28.54% | -12.64% | 20.18% | -14.45% | 3.38% |
| OTM (MT) | -27.41% | -7.37% | -49.95% | -15.79% | 38.82% | -10.80% | -4.17% |
| %MSE (MT) | 4.44% | 10.15% | 10.32% | 4.50% | 9.52% | 31.01% | 12.73% |
| ITM (LT) | -3.02% | 0.56% | -16.60% | -5.90% | 2.07% | 29.67% | 10.30% |
| NTM (LT) | -3.56% | -9.79% | -17.82% | -3.22% | 10.27% | 36.27% | 2.79% |
| OTM (LT) | -18.18% | -21.92% | -27.20% | -8.48% | 23.89% | 24.85% | -8.79% |
| %MSE (LT) | 3.51% | 13.75% | 16.28% | 3.89% | 10.15% | 44.38% | 3.19% |

*Note*: ITM, NTM, and OTM are contractions for valuation results of in-the-money, near-the-money, and out-of the-money put options, respectively. ST-, MT-, and LT- are abbreviations for options with maturity of short term, medium term, and long term to expiration, respectively. For each category specified by the moneyness, the valuation errors are measured as the raw value of average relative valuation errors using the loss function of ${}^{1}\!/_{n}\sum_{t=1}^{n}(P_{i,t}-P_{i,t}^{M})/P_{i,t}^{M}$ to examine the over- and under-pricing of model performance; While for categories labelled by %MSE (percentage mean squared errors), the valuation errors are measured as ${}^{1}\!/_{n}\sum_{t=1}^{n}[(P_{i,t}-P_{i,t}^{M})/P_{i,t}^{M}]^{2}$ where $P_{i,t}$ and $P_{i,t}^{M}$ are the *i*th theoretical put option price and market put option price at time point *t*.

**Table 3.5c. Overall valuation results**

| | GARCH (3.9) | E-GARCH (3.10) | GJR-GARCH (3.11) | NA-GARCH (3.12) | HN-GARCH (3.13) | Black-Scholes (3.18) | Gram-Charlier (3.19) |
|---|---|---|---|---|---|---|---|
| %MSE Call | 1.63% | 3.89% | 4.50% | 1.95% | 13.70% | 28.03% | 3.59% |
| %MSE Put | 4.43% | 11.06% | 13.28% | 4.29% | 12.82% | 35.59% | 8.19% |
| %MSE Overall | 3.03% | 7.47% | 8.89% | 3.12% | 13.26% | 31.81% | 5.89% |
| K-S test | 0.044 | 0.027 | 0.04 | 0.037 | 0.033 | 0.024 | 0.027 |

Percentage mean squared errors (%MSE) are computed respectively for S&P500 call, put, and the overall option sample. One-sample Kolmogorov-Smirnov test is used to examine the robustness of valuation results, with the null hypothesis that the valuation errors of the given model come from the normal distribution, against the alternative hypothesis that it deviates from such distribution.

In sum, with aggregated call and put valuation results shown in Table 3.5c, the simple GARCH model has the best performance, next is the NA-GARCH, and the Gram-Charlier model comes third in the overall valuation aspect. This finding is somewhat below our expectation, as the option models should have virtually identical performance for either the call or put valuation in light of the put-call parity. However, in case for any changes occur in market conditions, the supply and demand for the call and put options will differ from others and consequently affect the call and put option price differently (Cremers and Weinbaum 2010). In a related study of Chiang and Huang (2011), market momentum is taken into consideration when they evaluate the option price. They suggest that the simple GARCH performs best during the upward moving economies while the exponential GARCH model performs well during an economic downturn, which seem similar to the findings here.

With respect to the NA-GARCH's valuation results, its performance is consistent with Christoffersen and Jacobs (2004b), Hsieh and Ritchken (2005), Christoffersen et al (2010) who support the use of GARCH models in the non-linear asymmetric form. Also, in contrast to most previous GARCH option pricing exercises, the Gram-Charlier model performs more satisfactorily than the remaining models, including the HN-GARCH and E-GARCH model. One potential reason can be attributed to its accuracy in pricing the out-of-the-money put options and in-the-money call options, which stress the importance of characterising the third and fourth moment of underlying asset price distribution. And the valuation performance of the E-GARCH is noticeable as well. In spite of its higher valuation errors than the NA-GARCH, the E-GARCH model dominates the remaining models particularly under the short-term maturity category, which is in line with the finding of Stentoft (2005) and Byun and Min (2013). Potential reasons could be that the E-GARCH has a news impact function which is more sensitive to the small magnitude change from the information shock. Finally, as the K-S test statistic values indicate, all valuation results show normal distribution at 5% significance level, which reject the alternative hypothesis that valuation errors deviate from the normal distribution.

## 3.5 Conclusion

In this chapter a range of parsimonious GARCH models are examined as instruments to determine the S&P500 option price. The valuation results indicate that the non-linear asymmetric GARCH model continue to be the preferred leverage GARCH for option pricing. However, in view of the results from both the S&P500 call and put valuation, the basic GARCH model dominates our pricing tournament with the least pricing errors in our exercise. This result rejects our earlier hypothesis that leverage GARCH models outperform simple GARCH model. Following the basic GARCH and NA-

GARCH models, the third model comes to the Gram-Charlier formula, suggesting the benefits of accommodating the third and fourth moment of underlying price distribution in the option pricing process.

These findings have important implications. Although there exist numerous developed models for the option pricing, there are hurdles in their application due to model advances, sophistication, and ones' own numerical methods. On the contrary, for a study considering parsimonious models for effective option pricing remains void in the literature. Our exercise filled this gap by studying a range of commonly used GARCH models but give amendments to have their most parsimonious formulation. Thus these models can be easily estimated using typical statistical packages such as R, Eviews, S-plus and Stata, which is practical merit over other self-owned models in the literature.

Finally, using the Black-Scholes model and the Gram-Charlier models as benchmarks, our investigation into the comparative performances of the various GARCH variants is more rounded. Although the Black-Scholes model continuous to be favoured owing to its simplicity and tractability, it is safe to predict that in the field of empirical option research, numerical methods will be used increasingly in the future. As computer getting faster at a remarkable pace, the need for analytically tractable models will tend to diminish as alternative numerical methods become more open source, although expositional barriers still remain. For future research it would be interesting to examining these models' performance based on alternative distributions and investigate the option valuation performance.

**Chapter 4**

# Foreign exchange option pricing using the persistent and transitory component GARCH model☆


## Abstract

This chapter investigate the performance of the component GARCH model in modelling the foreign exchange rate volatilities and pricing the foreign exchange options. Our assessment finds additional findings of short lived volatility asymmetry in the EURUSD and GBPUSD exchange rate. The component GARCH model gives overall improved performance in the valuation of the foreign exchange options, particularly for the contracts with the long term to expiration.



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## 4.1 Introduction

Recap the previous chapter that singular volatility component GARCH models give reasonable performance in evaluating the option, but their valuation errors appear to increase with maturity. This chapter studies an alternative GARCH framework which allows for component volatility structure characterising both the short and long run dynamic of the foreign exchange volatility. As illustrated in Engle and Lee (1999), Christoffersen et al. (2008), and Christoffersen et al. (2012a), we allow for more persistent volatility dynamics in the non-linear GARCH model by including a long run volatility component to the conventional single volatility equation GARCH models. This framework allows for both transitory and persistent volatility evolution as well as the volatility asymmetries. The resulting formulations are estimated using the maximum likelihood method and the foreign exchange option are valued through the Monte Carlo simulation approach.

Four different but nested forms are considered in our study. The SGH (the simple form GARCH model), has the symmetric and single volatility component. The AGH (asymmetric form GARCH model), has the asymmetric and single volatility representation. The SCGH (symmetric component GARCH model), has the transient and persistent component volatility representation with symmetric return-volatility relationship. Lastly, the ACGH (asymmetric component GARCH model) accounts for return-volatility asymmetry and has both transient and persistent volatility components, which nests the SCGH models when return volatility linkage is symmetric, the AGH model when the volatility has singular volatility component representation, and the SGH when the SCGH is reduced to the single component form. These models are estimated using the exchange rate of EURUSD, GBPUSD, and GBPEUR with 20 years of daily returns. After estimating these specific forms, the risk-neutralised GARCH models are derived for the valuation of foreign exchange options.

The current study's empirical results emphasize the importance of persistent volatility component in describing the volatility character over the long term. The maximum likelihood results support the benefit of adding a long run component as measurement for the exchange rate volatility. When using the risk-neutralised GARCH models to evaluate foreign exchange options, the component GARCH models exhibit the overall improved performance. The asymmetric component model yields the least root mean absolute valuation errors in valuing the EURUSD and GBPEUR exchange rate. The symmetric component GARCH model also has satisfactory performance. In valuing the GBPUSD exchange options it yields the least root mean absolute valuation errors amongst the GARCH models. Finally, it is worthy to note the performance of the Garman and Kohlhagen model. When calibrating the model with the deterministic volatility functions, there is significant improvement in its valuing performance particularly in pricing the GBPUSD options, with a root mean absolute valuation error of 0.125 which outperforms other GARCH models.

The empirical analysis therefore supports the benefit of GARCH formulation with a component framework, at least in applying for the foreign exchange rate modelling and the foreign exchange options pricing. Although the existing literature on currency option valuation using GARCH models support the single volatility component GARCH models, the current study's findings suggest that a GARCH framework with volatility asymmetry and additional long run volatility component is crucial in the valuation of foreign exchange options.

The methodology used in the present study is closely related to the component GARCH model study of Engle and Lee (1999) and Christoffersen et al. (2008), who found significant improvement of component framework for volatility modelling in equities in S&P500 index returns. Our research finds similar evidence using the foreign exchange returns. More importantly, a comparative analysis is carried out among these specified GARCH models. Christoffersen et al. (2012a) empirically investigate the performance of the component GARCH model with affine restrictions. However, none of these studies considered the foreign exchange market. This study attempts to examine this significant omission and investigate whether a GARCH model accounting for a component formulation and volatility asymmetry yields an improvement when modelling the foreign exchange volatilities, and more importantly, evaluating the foreign exchange options.

The remainder of this chapter is organised as follows. Section 4.2 reviews the related literature and identifies the research gaps. Section 4.3 presents the GARCH models specified and the valuation procedures with risk-neutralised models derived. Section 4.4 presents our data with descriptive statistics. The model estimation results are also reported in this section. Section 4.5 reports the valuation performance of each GARCH model and tests the significance of the valuation results. Section 4.6 summarises important findings and contributions made from the current study.

## 4.2 Review of Related Studies

The earliest models proposed for pricing currency options are attributed to Garman and Kohlhagen (1983), and Biger and Hull (1983), which are developed from the classic Black and Scholes (1973) formula with the original assumptions retained. These models have long been criticised for their constant volatility assumption which can result in a significant deficiency, and requiring a stochastic volatility description to achieve further empirical success (Fouque et al. 2000; Bates 2003; Alexander 2008b; Christoffersen et al. 2012c).

Evaluating options with models that embody a precisely estimated stochastic volatility representation is theoretically a more robust approach but computationally is more demanding. In continuous-time models, [for example Heston (1993), Rosenberg (1998), Sarwar and Krehbiel (2000), Bollen and Rasiel (2003), Leung et al. (2013), Shokrollahi and Kilicman (2014)], the volatilities are assumed unobservable and the frequently updated implied volatilities must be used. This approach involves model parameters remaining unchanged through time, and computing numerous implied volatilities from the traded options, one for each strike price and for every maturity date. This becomes computationally burdensome as the amount of market option data increase. Also in case of thinly or illiquid markets the contemporaneous options records may not always be sufficient and reliable, which substantially impedes the use of implied volatilities when valuing option price (Heston and Nandi 2000).

Unlike filtering a volatility variable from discrete observations thorough continuous time option models, the GARCH models provide an inherent advantage given the observable time-varying volatilities from historical underlying prices. Therefore one could directly estimate a precise volatility variable from the historical asset returns instead of expensive calculation for the implied volatilities from previously traded options. The rich variety of the GARCH variants also enables one to further characterise the volatility representation so as to better facilitate an accurate option pricing exercise.

Related studies valuing currency options using GARCH models include Duan and Wei (1999), Bollen and Rasiel (2003), Harikumar et al. (2004), Posedel (2006), Irena (2009), Manzur et al. (2010), Aduda (2011), Gozgor and Nokay (2011), Ulusoy and Onbirler (2014), and Bhat and Arekar (2016) to date. Of these papers, evidence is presented and suggests that the GARCH model fits the foreign exchange distribution better than the Brownian model. However, an assessment concerning suitable GARCH framework for valuing currency options remains largely unexplored in these studies.

Studies by Carr and Wu (2007), Bakshi et al. (2008), and Leung et al. (2013), find that volatility asymmetry exists in the foreign exchange rate market due to the noise traders and hedging behaviours through currency options, which restricts not only downside exposure but allows upside potential. On the other hand, Hsieh (1988), Bollerslev et al. (1992), Andersen et al. (2001), and Maya and Gomez (2008), posit that foreign exchange markets have virtually more symmetric linkage between its price movement and volatility. These studies claim that in the case of a foreign currency depreciations causing negatives returns to the foreign currency holder, it results in positive proceeds for the domestic currency holders.

The present work is also concerned with the component GARCH formulation in foreign exchange volatility modelling and corresponding currency option pricing, which employ an additional long run volatility component for the volatility description. Our application using the component GARCH model for the foreign exchange rate is motivated from the deterministic theory of foreign exchange market. It is well documented that foreign exchange market is actively traded (Dornbusch 1976). Therefore when new information arrives, in the short run currency price movement tend to respond more sharply but accompanied with quick time decay (Rogoff 1996). However, in light of the law of one price and purchasing power parity, in the long run it is price level differences between two currencies that determine long term exchange rate movements (Kilian and Zha 2002). These observations suggest that a GARCH framework with short- and long run volatility components may better describe exchange rate volatility.

There are several studies modelling the volatility persistence over long time horizon, such as the fractionally integrated GARCH models of Baillie et al. (1996) and the component GARCH given in Engle and Rosenberg (1995) and in Engle and Lee (1999). Christoffersen et al. (2008) generalise the component GARCH model for option valuation, with the affined structure proposed to yield the quasi-analytical solution. Wang (2008) compares the performance of a component GARCH model and fractionally integrated GARCH model, and concludes that the former is preferred since the later artificially prolongs the leverage effect which result from the fractional integration. Dziubinski (2011) simplifies the GARCH model of Christoffersen et al. (2008) and suggests that simplified model could also give satisfactory performance. The component GARCH models with and without affined restriction are summarised in Christoffersen et al. (2012a), in which model performance of measuring S&P500 returns and pricing S&P500 options are examined. By and large, these studies conduct important assessments of component GARCH models, but an empirical inquiry using the component GARCH model for the currency option valuation remain largely undeveloped. This study aims to fill this gap.

### 4.3 The Models and Valuation procedure

#### 4.3.1 The GARCH models

The currency option pricing models of Duan and Wei (1999), in its physical measure[12] P, can be written with following equations,

$$\ln(X_{t+1} / X_t) = r_{t+1,\tau,d} - r_{t+1,\tau,f} + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \varepsilon_{t+1}, \tag{4.1}$$

$$\varepsilon_{t+1} \mid \mathcal{F}_{t+1} \sim N\left(0,\ h_{t+1}\right),$$

$$h_{t+1} = \omega + \beta_1 h_t + \alpha_1\left(\varepsilon_t - \gamma\sqrt{h_t}\right)^2, \tag{4.2}$$

The first equation (4.1) is conditional mean equation in which the risk premium $\lambda$ is multiplied by the conditional standard deviation and that the conditional volatility enters the mean equation to affect the exchange rate movement. For the remaining coefficients, $X_t$ denotes the foreign exchange rate at time *t*, $r_{t+1,\tau,d}$ and $r_{t+1,\tau,f}$ denote the domestic and foreign daily risk free rate, $\lambda$ denotes the foreign exchange risk premium. $\varepsilon_{t+1}$ denotes the unexpected return

[12] The physical measure refers to that the derivatives price is the discounted value of their future payoff which is proportional to the risk premium of the underlying asset (McDonald, 2011).

with zero mean and variance of $h_{t+1}$ , $\mathcal{F}_t$ denotes the information set at time point t. $\gamma$ describes the asymmetry magnitude and is linked by $\alpha_1$ to affect the conditional volatility. In case that $\gamma$ equals zero, the conditional volatility equation in (3.2) reduces to its symmetric form,

$$h_{t+1} = \omega + \beta_1 h_t + \alpha_1 \varepsilon_t^2 \quad . \tag{4.3}$$

As illustrated in Christoffersen et al. (2012a), the asymmetric component model composed of short and long run volatility components can be written as the following,

$$h_{t+1} - q_{t+1} = \beta\left(h_t - q_t\right) + \alpha\left(\varepsilon_t^2 - 2\gamma_1\varepsilon_t\sqrt{h_t} - h_t\right), \tag{4.4}$$

$$q_{t+1} = \omega + \rho q_t + \varphi\left(\varepsilon_t^2 - 2\gamma_2\varepsilon_t\sqrt{h_t} - h_t\right). \tag{4.5}$$

in which the volatility component, i.e. $h_{t+1} - q_{t+1}$, describes the transient volatility dynamic over the short term and the other component $q_{t+1}$ that characterises the persistent volatility behaviour over a long time horizon. The parameters in (4.4) and (4.5) are constrained to be $0 \le \alpha < \beta < 1$, $\alpha + \beta < \rho < 1$ and $0 < \varphi < \beta$, which are imposed to ensure that volatility evolution over the short- and long run remains mean-reverting for all time points with the probability of one. The information shock of volatility in the short run is more sensitive to the temporary information shock and has a more swift time decay approaching the unconditional volatility. In contrast, the long run volatility process of $q_t$ evolves with greater persistence and therefore has a slower mean-reversion rate. To accommodate the potential volatility asymmetries, both $h_{t+1} - q_{t+1}$ and $q_{t+1}$ are functions of volatilities asymmetry parameters. Two parameters, $\gamma_1$ and $\gamma_2$ are considered here given that the asymmetric relations between exchange rate return and volatility changes may have transitory and persistent impact. In case that $\gamma_1$ and $\gamma_2$ have value of zero the component GARCH model is reduced to its symmetric form,

$$h_{t+1} = q_{t+1} + \beta\left(h_t - q_t\right) + \alpha\left(\varepsilon_t^2 - h_t\right), \tag{4.6}$$

$$q_{t+1} = \omega + \rho q_t + \varphi\left(\varepsilon_t^2 - h_t\right). \tag{4.7}$$

| **Table 4.1 The GARCH models under measure P** |
|---|
| The symmetric GARCH-in-Mean model (SGH) |
| $\ln(X_{t+1} / X_t) = r_{t+1,\tau,d} - r_{t+1,\tau,f} + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \varepsilon_{t+1},$ <br> $h_{t+1} = \omega + \beta_1 h_t + \alpha_1 \varepsilon_t^2$ . |
| The asymmetric GARCH-in-Mean model (AGH) |
| $\ln(X_{t+1} / X_t) = r_{t+1,\tau,d} - r_{t+1,\tau,f} + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \varepsilon_{t+1},$ <br> $h_{t+1} = \omega + \beta_1 h_t + \alpha_1\left(\varepsilon_t - \gamma\sqrt{h_t}\right)^2.$ |
| The symmetric component GARCH-in-mean model (SCGH) |
| $\ln(X_{t+1} / X_t) = r_{t+1,\tau,d} - r_{t+1,\tau,f} + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \varepsilon_{t+1},$ <br> $h_{t+1} = q_{t+1} + \beta\left(h_t - q_t\right) + \alpha\left(\varepsilon_t^2 - h_t\right),$ <br> $q_{t+1} = \omega + \rho q_t + \varphi\left(\varepsilon_t^2 - h_t\right).$ |
| The asymmetric component GARCH-in-Mean model (ACGH) |
| $\ln(X_{t+1} / X_t) = r_{t+1,\tau,d} - r_{t+1,\tau,f} + \lambda\sqrt{h_{t+1}} - \frac{1}{2}h_{t+1} + \varepsilon_{t+1},$ <br> $h_{t+1} = q_{t+1} + \beta\left(h_t - q_t\right) + \alpha\left(\varepsilon_t^2 - 2\gamma_1\varepsilon_t\sqrt{h_t} - h_t\right),$ <br> $q_{t+1} = \omega + \rho q_t + \varphi\left(\varepsilon_t^2 - 2\gamma_2\varepsilon_t\sqrt{h_t} - h_t\right).$ |

Table 4.1 presents the GARCH models respectively specified with symmetry, asymmetry, singular, and component setup. Therefore for any occurrence of short, long run volatility asymmetry and persistence can be explicitly assessed. Notice also that the four GARCH specifications are implicitly related with each other. To see this, for the asymmetric GARCH model (AGH) when its asymmetry parameter $\gamma$ becomes zero then it is reduced to the symmetric GARCH model. The component GARCH model of SCGH and ACGH model actually further specifies the long run and persistence nature of volatility which was originally defined by $\omega$ in the SGH and AGH model, and this long run volatility persistence parameter is represented by the volatility equations $q_t$ in the SCGH and ACGH models. Also as aforementioned before when $\gamma_1$ and $\gamma_2$ take value of zero then the asymmetric component GARCH model is reduced to the symmetric component GARCH models.

### 4.3.2 The Volatility Term Structure

Both conventional GARCH and component GARCH models describe the persistence feature of the volatility dynamic. However, it is expected that component GARCH model will provide a more effective assessment of long run conditional volatility. To see this, we follow Christoffersen et al. (2012a) and derive the volatility term structure of both models so as to inspect their mean-reversion rate over time.

With the conditional variance equation in the asymmetric non-linear form,

$$h_{t+1} = \omega + \beta_1 h_t + \alpha_1 \left( \varepsilon_t - \gamma \sqrt{h_t} \right)^2 .$$

We have the expectation for the conditional variance as

$$E\left[h_{t+1}\right]=\omega+(\alpha_1(1+\gamma^2)+\beta_1)\sigma^2, \tag{4.8}$$

where $\sigma^2$ denotes the unconditional variance. Then the two-step ahead expectation $E\left[h_{t+2}\right]$ can be written as:

$$E\left[h_{t+2}\right]=\omega+\omega(\alpha_1(1+\gamma^2)+\beta_1)+(\alpha_1(1+\gamma^2)+\beta_1)^2\sigma^2$$

and by induction we have the *n*-step ahead variance term structure,

$$E\left[h_{t+n}\right]=\frac{1-(\alpha_1(1+\gamma^2)+\beta_1)^n}{1-(\alpha_1(1+\gamma^2)+\beta_1)}\omega+(\alpha_1(1+\gamma^2)+\beta_1)^n\sigma^2. \tag{4.9}$$

With regard to the component GARCH model, given its conditional volatility equation taking the expectation for (4.4) and (4.5) with $E[\varepsilon_t^2]=h_t$ and $E[\varepsilon_t]=0$, we have,

$$\begin{aligned}
E[h_t]&=q_t+\beta(h_{t-1}-q_{t-1}),\\
E[q_t]&=\omega+\rho q_{t-1}
\end{aligned}$$

Then we have the n-step ahead expectation respectively for the transitory and persistence volatility component,

$$\begin{aligned}
E[h_{t+n}]&=q_{t+n}+\beta^n(h_t-q_t)\\
E[q_{t+n}]&=\left(\frac{1-\rho^n}{1-\rho}\right)\omega+\rho^n q_t
\end{aligned}$$

Replacing $q_{t+n}$ by $E[q_{t+n}]$ yields:

$$E[\sigma_{t+n}^2]=\left(\frac{1-\rho^n}{1-\rho}\right)\omega+\rho^n q_t+\beta^n(h_t-q_t)\,, \tag{4.10}$$

which enables us to examine the strength of persistence of each model.

### 4.3.3 The risk neutral transformation

As shown in Duan and Wei(1999), the GARCH models expressed in their physical probability measure have to be cast in their risk-neutral forms to obtain the option value. This derivation involves the use of two-economy state local risk-neutral valuation relationship. Specifically, for any foreign exchange rate given price $X_{t,d}$ at time *t* in domestic currency d, (i) the return $X_{t,d} / X_{t-1,d}$ has the lognormal distribution under the risk-neutral measure Q, (ii) $E^{Q}(X_{t,d} / X_{t-1,d})$ yields a domestic risk-free rate return $r_{t+1,\tau,d}$ almost surely under measure P, and (iii) $Var^{Q}(X_{t,d} / X_{t-1,d} \mid \mathcal{F}_t) = Var^{P}(X_{t,d} / X_{t-1,d} \mid \mathcal{F}_t)$ with an unexpected return defined as $\xi_{t+1} \mid \mathcal{F}_t \sim N(0, h_{t+1})$, where $\xi_{t+1} = \varepsilon_{t+1} + \lambda\sqrt{h_{t+1}}$ .

Table 4.2 summarises risk-neutralised GARCH models for option valuation, including the standard GARCH-in-mean model (SGH), asymmetric GARCH-in-mean model (AGH), the symmetric component GARCH-in-mean model (SCGH), and finally the asymmetric component GARCH-in-mean model (ACGH). Therefore in this setup a relatively explicit comparative analysis can be conducted, which are the standard- versus component GARCH and symmetric- versus the asymmetric model.

| Table 4.2 The GARCH models under measure Q |
|---|
| The symmetric GARCH-in-Mean model (SGH) |
| $\ln(X_{t+1}/X_t) = r_{t+1,\tau,d} - r_{t+1,\tau,f} - \frac{1}{2}h_{t+1} + \xi_{t+1},$<br>$h_{t+1} = \omega + \beta_1 h_t + \alpha_1(\xi_t - \lambda\sqrt{h_t})^2.$ |
| The asymmetric GARCH-in-Mean model (AGH) |
| $\ln(X_{t+1}/X_t) = r_{t+1,\tau,d} - r_{t+1,\tau,f} - \frac{1}{2}h_{t+1} + \xi_{t+1},$<br>$h_{t+1} = \omega + \beta_1 h_t + \alpha_1(\xi_t - \lambda\sqrt{h_t} - \gamma\sqrt{h_t})^2.$ |
| The symmetric component GARCH-in-mean model (SCGH) |
| $\ln(X_{t+1}/X_t) = r_{t+1,\tau,d} - r_{t+1,\tau,f} - \frac{1}{2}h_{t+1} + \xi_{t+1},$<br>$h_{t+1} = q_t + \beta(h_t - q_t) + \alpha[(\xi_t - \lambda\sqrt{h_t})^2 - h_t],$<br>$q_{t+1} = \omega + \rho q_t + \phi[(\xi_t - \lambda\sqrt{h_t})^2 - h_t].$ |
| The asymmetric component GARCH-in-mean model (ACGH) |
| $\ln(X_{t+1}/X_t) = r_{t+1,\tau,d} - r_{t+1,\tau,f} - \frac{1}{2}h_{t+1} + \xi_{t+1},$<br>$h_{t+1} = q_{t+1} + \beta(h_t - q_t) + \alpha[(\xi_{t-1} - \lambda\sqrt{h_t})^2 - 2\gamma_1(\xi_t - \lambda\sqrt{h_t}) - q_t],$<br>$q_{t+1} = \omega + \rho q_t + \phi[(\xi_t - \lambda\sqrt{h_t})^2 - 2\gamma_2(\xi_t - \lambda\sqrt{h_t}) - h_t].$ |

### 4.3.4 Simulating the GARCH model option price

The Monte Carlo simulation was applied to calculate the GARCH model[13] option price. As an illustration, consider the asymmetric GARCH model under measure *Q* used by Duan and Wei (1999),

[13] For our current empirical analysis, only the single lag GARCH structure is considered in much of the paper. Literature on the subject suggests that this single lag structure reconciles the discrete-time GARCH framework with the continuous-time option model to financial option valuation (Duan, 1997; Heston and Nandi, 2000).

$$
\ln(X_{t+1}/X_t) = r_{t+1,\tau,d} - r_{t+1,\tau,f} - \frac{1}{2}h_{t+1} + \xi_{t+1},
$$
$$
h_{t+1} = \omega + \beta_1 h_t + \alpha_1(\xi_t - \lambda\sqrt{h_t} - \gamma\sqrt{h_t})^2.
$$

At the end of date *t* we have the conditional variance $h_{t+1}$ thus we can obtain the ln($X_{t+1}/X_t$) though the conditional mean equation. Note that under the risk-neutral measure the unexpected return $\xi_t = v_t\sigma_t$ where $v_t$ is distributed as $N(0,1)$ and can be obtained from pseudo random number generators. Then we simulate the conditional variance at time point *t+2,*

$$
h_{t+2} = \omega + \alpha_1 h_{t+1}\left(\xi_{t+1} - \lambda\sqrt{h_{t+1}} - \gamma\sqrt{h_{t+1}}\right)^2 + \beta_1 h_{t+1}
$$

For each round simulation of the conditional variance $h_{t+i}$, we then simulate the exchange rate level through the conditional mean equation. This procedure is implemented sequentially until the simulation reaches the pre-specified expiration date. In the meanwhile, we aggregate the underlying asset returns to arrive at its terminal value $R_{i,t+\tau}$ at the maturity date $t+\tau$. For i=1,…MC[14] individual simulation:

$$
\begin{array}{ccccccccccccccc}
v_{1,1} & \rightarrow & h_{1,t+2} & \rightarrow & R_{1,t+2} & \cdots & \rightarrow & v_{1,\tau} & \rightarrow & h_{1,t+\tau} & \rightarrow & R_{1,t+\tau} & & & \\
v_{2,1} & \rightarrow & h_{2,t+2} & \rightarrow & R_{1,t+2} & \cdots & \rightarrow & v_{2,\tau} & \rightarrow & h_{2,t+\tau} & \rightarrow & R_{2,t+\tau} & \searrow & & \\
 & & \cdots & & & & & & & \cdots & & & \rightarrow & \overline{R_T} \\
 & & \cdots & & & & & & & \cdots & & & \nearrow & & \\
v_{MC,1} & \rightarrow & h_{MC,t+2} & \rightarrow & R_{MC,t+2} & \cdots & \rightarrow & v_{MC,\tau} & \rightarrow & h_{MC,t+\tau} & \rightarrow & R_{MC,t+\tau} & & &
\end{array}
$$

[14] 20000 times simulation are used in our valuation practice.

Taking the average value of the simulated exchange rate return from $R_{1,t+\tau}$ to $R_{MC,t+\tau}$, we obtain the terminal price of exchange rate $X_T$,

$$X_T = X_t \exp\left\{(T-t)\left(r_{t,d} - r_{t,f}\right) - \frac{1}{2}\sum_{i=1}^{\tau}\sigma_{t+\tau}^2 + \sum_{i=1}^{\tau}\xi_{i+\tau}\right\}$$

The currency call, $C_t^{FX}$ issued at date t with strike price K, and time to expiration $\tau = T-t$, equals to

$$C_t^{FX} = \exp(-r_{t+1,\tau,d}\tau)E^Q\left[\max\left(0, X_T - K\right)\right] \tag{4.11}$$

### 4.3.5 The benchmark model

We employ the Garman and Kohlhagen (1983) model as the benchmark in comparative analysis, which is developed from the Black-Scholes (1973) model with amendments to the foreign and domestic interest rate and interest rate parity. To express the model, letting $r_{t,\tau,d}$ be the domestic risk-free rate, $r_{t,\tau,f}$ be the foreign risk-free rate at time-*t*, τ and σ be respectively the maturity and the annual standard deviation, and X and K the respective underlying exchange rate and exercise price, then the Garman and Kohlhagen (1983) (henceforth G-K) model can be written as,

$$\begin{aligned} C &= \exp\left(-r_{t,\tau,f}\tau\right)X_t N\left(d_1\right) - K\exp\left(-r_{t,\tau,d}\tau\right)N\left(d_2\right), \\ d_1 &= \left[\ln\left(X_t / K\right) + \left(r_{t,\tau,d} - r_{t,\tau,f} + \frac{1}{2}\sigma^2\right)\tau\right] / \sigma\sqrt{\tau}, \\ d_2 &= \left[\ln\left(X_t / K\right) + \left(r_{t,\tau,d} - r_{t,\tau,f} - \frac{1}{2}\sigma^2\right)\tau\right] / \sigma\sqrt{\tau}. \end{aligned} \tag{4.12}$$

The original form of the Garman and Kohlhagen model assumes instantaneously constant volatilities. To remedy this deficiency we implement the deterministic volatility function introduced in Dumas et al. (1998) and in Christoffersen and Jacobs (2004b) to characterise implied volatility patterns across exercise price and time to maturity, and fit estimated volatility to the Garman-Kohlhagen model to obtain model option price. Although this procedure is inconsistent with the original constant volatility assumption, it is a variation of what is operated by practitioners who smooth the implied volatilities and calculate the predicted option price (Dumas et al. 1998).

To implement the deterministic volatility function, we first estimate the implied volatility $\sigma_{IV,i}$ from market options,

$$\sigma_{IV,i} = Arg\min((C_i^{GK} - C_i^{Market}) / C_i^{Market})^2, \tag{4.13}$$

which is obtained by minimising the relative valuation error loss function. With the obtained implied volatility series $\sigma_{IV}$ we estimate the following equation which specifies volatilities in relation to the exercise price *K* and time to maturity *T*,

$$\sigma_{IV} = \theta_0 + \theta_1 K + \theta_2 K^2 + \theta_3 T + \theta_4 T^2 + \theta_5 KT + \varepsilon_{iv} \tag{4.14}$$

With the estimated parameters set $\theta(i)$ at time *t*, we forecast the implied volatility of the next working day as,

$$\sigma\ (\theta) = \theta_0 + \theta_1 K + \theta_2 K^2 + \theta_3 T + \theta_4 T^2 + \theta_5 KT\ , \tag{4.15}$$

and fit $\sigma(\theta)$ to formula (4.12) to have the predicted option price. Notice that in (4.14) and (4.15) the deterministic volatility functions are in quadratic form thus to characterise the parabolic shape of implied volatilities across exercise price.

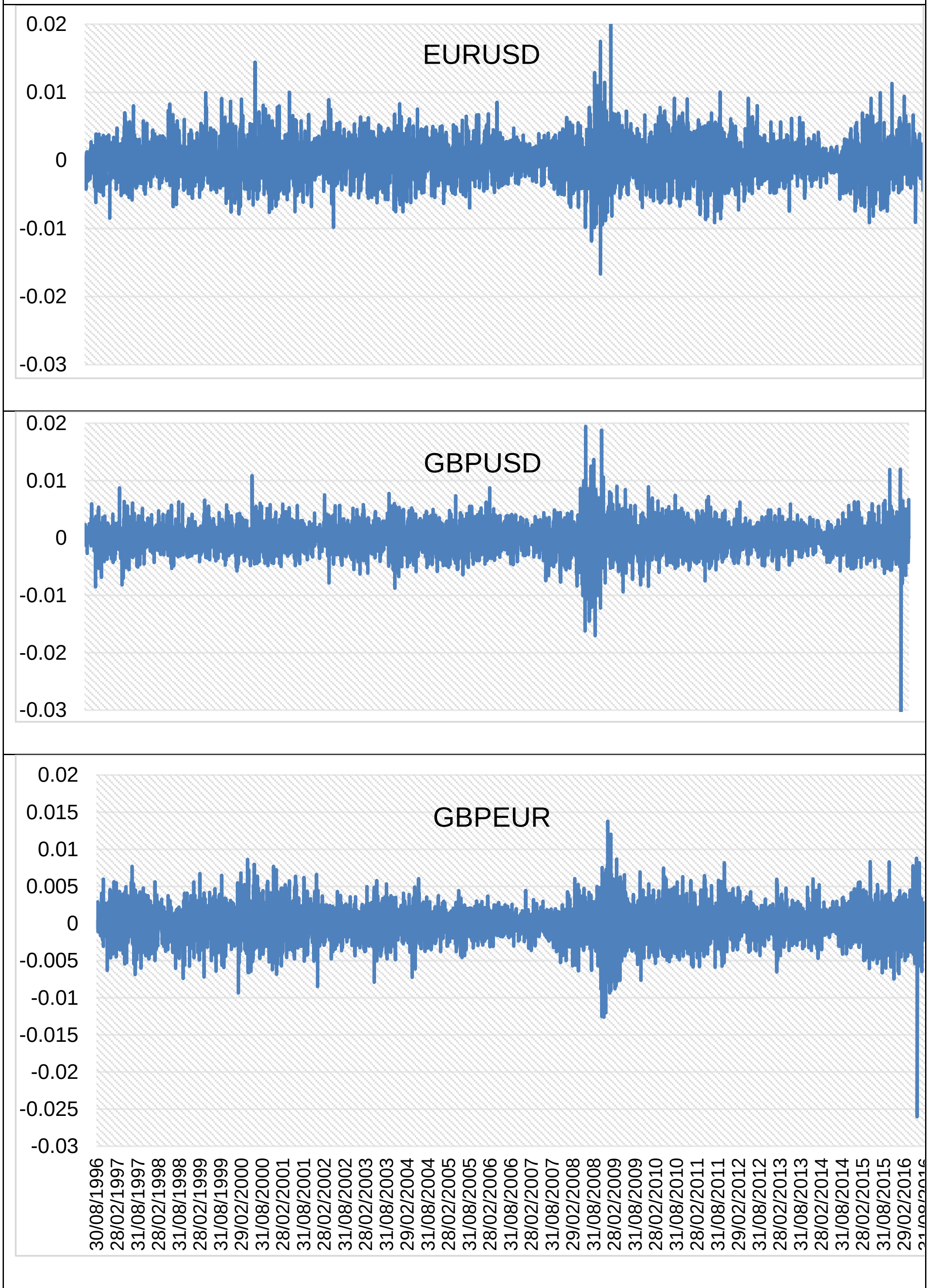
### 4.4 Data and Model Estimation

We used three sets exchange rates, the EURUSD, GBPUSD, and GBPEUR for the period 01-Sep-1996 and 31-Aug-2016 to estimate the models. The pricing units for three exchange rates are USD per 100 Euros, USD per 100 British Pounds, and EUR per 100 British Pound, respectively. We also used currency options written on these exchange rate for the period 01-June-2015 to 17-August-2016 to examine selected model option pricing performance. The currency options issued by Euronext Amsterdam were used in our exercise. The market is the second largest foreign exchange options market with an open interest second to the Chicago Mercantile Exchange. The underlying asset is the spot exchange rate rather than a futures price. The options are strictly European rather than American, therefore the currency options in Euronext do not have an early-exercise feature as CME options complicating the valuation exercise.

The historical price for both the underlying exchange rate and their foreign exchange options are obtained from Thomson Reuters Datastream. In line with Bakshi et al. (1997), Heston and Nandi (2000), and many others, for ease of computation we use the mid-value between the last reported bid ask option price for the market FX option data. For the domestic and foreign interest rates, we used the London Interbank Offered Rate (LIBOR) settled for each currency.

Figure 4.1 plots the daily exchange rate returns for three currencies over the sample period. The volatility of all three exchange rates appear time-varying and the amplitude appears more severe between 2008 and 2009, which is consistent with market movements during the 2007-2008 U.S. financial crisis and subsequent European debt crisis in 2009.

**Figure 4.1 Exchange Returns of three currency pairs**




The logarithmic returns of historical exchange rate between European Euro to United States Dollar (top), British Pound Sterling to US Dollar(middle), and British pound to European Euro (below) from 30/08/1986 until 30/08/2016 with 5219 observations.

Also around the first half of 2016 there are significant decreases on GBPUSD and GBPEUR exchange rate, which potentially is attributed to the withdrawal of the United Kingdom from European Union. The three exchange rate returns also exhibit a common pattern in which a small increase is followed by a small decrease, and a large increase is followed by a large decrease, indicating the volatility clustering effect.

Table 4.3 summarises the descriptive statistics for three exchange rates. All three currency pairs show a certain skewness with EURUSD positively skewed whereas GBPUSD and EURGBP negatively skewed. GBPUSD exhibits the highest value of kurtosis which implies a fat tail distribution. For all three exchange rates the ARCH effect tests producing the F-statistic values all greater than the critical values 5.992 from the $\chi^2$ distribution with 2 degree of freedom, accept the alternative hypothesis of conditional heteroscedasticity. Jarque-Bera test results reject the normality for all three currency pairs. The range of the returns indicate that the returns of GBPUSD exhibit the largest variation, which are greater than both the EURGBP and EURUSD exchange rate.

**Table 4.3 Descriptive statistics for exchange rate returns**

| | EURUSD | GBPUSD | GBPEUR |
|---|---|---|---|
| Mean | -1.312E-05 | -1.343E-05 | -2.902E-06 |
| Median | 0.00E+00 | 1.088E-05 | 5.919E-05 |
| Minimum | -0.017 | -0.036 | -0.026 |
| Maximum | 0.020 | 0.019 | 0.014 |
| Standard Deviation | 0.003 | 0.003 | 0.002 |
| Sample Variance | 7.103E-06 | 6.254E-06 | 5.150E-06 |
| Skewness | 0.186 | -0.601 | -0.395 |
| Kurtosis | 2.410 | 11.977 | 5.215 |
| ARCH effect Test | 121.25 | 137.55 | 217.148 |
| Jarque-Bera Test | 1289.6 | 31442 | 36651 |
| Range | 0.037 | 0.056 | 0.040 |

Table 4.4 summarises the model estimating results[15] with three exchange rates. Under each exchange rate category, the parameter estimates are displayed according to model specifications in Table 4.2 and t-values of estimates are reported in parentheses. The maximum likelihood value in estimating each model is given in the end row. The likelihood values indicate the improvement of the component model over the conventional single component model, and the asymmetric component model gives further improvement over the symmetric component model in fitting the exchange rate returns. The negative values of risk unit parameter $\lambda$, although not significant among all the estimated models, suggest a potential negative premium in response to exposures of any foreign exchange variation.

The effect from information shock to volatility in the short and long run are reflected from the estimated values of $\alpha$ and $\varphi$ with their t-statistics. Comparing these two estimated parameters across models, the shock effect on volatility evolution is more pronounced in the GBPUSD and GBPEUR exchange rates, but less significant in the EURUSD. With respect to the mean-reversion feature of each volatility dynamic, in examining this effect under the component model it can be seen that under both symmetric and asymmetric models $(\alpha + \beta)$ always have the highest value under the GBPUSD exchange rate, and the lowest under the GBPEUR exchange rate, indicating a more transit effect of the new information to the GBPEUR exchange rate volatility.

[15] We use the MATLAB optimization function fminsearch for all the GARCH parameters estimation.

## Table 4.4 GARCH model estimating results

| | EURUSD | | | | GBPUSD | | | | GBPEUR | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | SGH | AGH | SCGH | ACGH | SGH | AGH | SCGH | ACGH | SGH | AGH | SCGH | ACGH |
| $\omega$ | 9.76E-08 | 9.13E-08 | 1.88E-05 | 1.83E-05 | 1.96E-07 | 1.96E-07 | 2.04E-05 | 4.33E-07 | 9.60E-08 | 9.16E-08 | 2.08E-5 | 2.30E-05 |
| | (1.856) | (1.927) | | | (2.105) | (1.353) | | | (2.456) | (1.12) | | |
| $\lambda$ | -0.157 | -0.107 | -0.14 | -0.05 | -0.167 | -0.033 | -0.183 | -0.181 | -0.074 | -0.049 | -0.122 | -0.072 |
| | (-2.238) | (-0.945) | (-5.374) | (-25.73) | (-2.087) | (-2.042) | (-1.462) | (-2.602) | (-1.341) | (-0.64) | (1.36) | (-0.387) |
| $\alpha$ | 0.029 | 0.028 | 1.42E-9 | 1.49E-5 | 0.045 | 0.045 | 0.042 | 7.02E-5 | 0.045 | 0.045 | 0.036 | 0.031 |
| | (8.021) | (9.433) | (8.16E-9) | (0.645) | (6.335) | (4.633) | (0.763) | (3.161) | (8.472) | (4.14) | (6.6) | (2.034) |
| $\beta$ | 0.969 | 0.969 | 0.615 | 0.432 | 0.949 | 0.949 | 0.899 | 0.5 | 0.953 | 0.953 | 0.04 | 0.053 |
| | (261.169) | (238.716) | (0.187) | (37.226) | (104.81) | (68.950) | (34.498) | (3.449) | (166.476) | (35.53) | (1.67) | (0.531) |
| $\gamma$ | | -0.064 | | | | -0.129 | | | | -0.022 | | |
| | | (-0.728) | | | | (-2.191) | | | | (-1.63) | | |
| $\rho$ | | | 0.997 | 0.996 | | | 0.996 | 0.996 | | | 0.996 | 0.996 |
| | | | (19.13) | (486.375) | | | (95.050) | (429.178) | | | (341.639) | (724.351) |
| $\varphi$ | | | 0.029 | 0.029 | | | 0.016 | 0.046 | | | 0.04 | 0.042 |
| | | | (0.661) | (17.441) | | | (4.129) | (8.217) | | | (1.516) | (8.836) |
| $\gamma_1$ | | | | -2.751 | | | | -2.24 | | | | -3.17E-03 |
| | | | | (-136.812) | | | | (-6.503) | | | | (-1.215) |
| $\gamma_2$ | | | | -6.5E-4 | | | | 2.52E-4 | | | | -1.34E-04 |
| | | | | (-1.715) | | | | (0.384) | | | | (-2.25E-3) |
| Likelihood | 19366.43 | 19366.34 | 19366.49 | 19366.99 | 19927.16 | 19927.38 | 19932 | 19933 | 20368.69 | 20368.59 | 20371.45 | 20373.27 |

The table summarise the estimating results using three exchange rate pairs for all four GARCH models given by Table 4.2. SGH, AGH refer to the standard GARCH model and asymmetric GARCH model. SCGH and ACGH refer to the symmetric component GARCH model and the asymmetric component GARCH model. All three exchange rates, i.e. the EURUSD, GBPUSD, and EURGBP, have time span between 01-Jan-1996 and 1-Aug-2016 with 5130 observations. t-values are reported in parentheses. Maximised likelihood values in model estimation are reported at the end of each column.

However, under the GBPUSD exchange rate, this impact in comparison is more persistent. Additionally, in examining the values of $(\alpha+\beta)$ of component models, this sum is always smaller than $\rho$ in their long run volatility component, corroborating our earlier model condition that *α+β<ρ<1*.

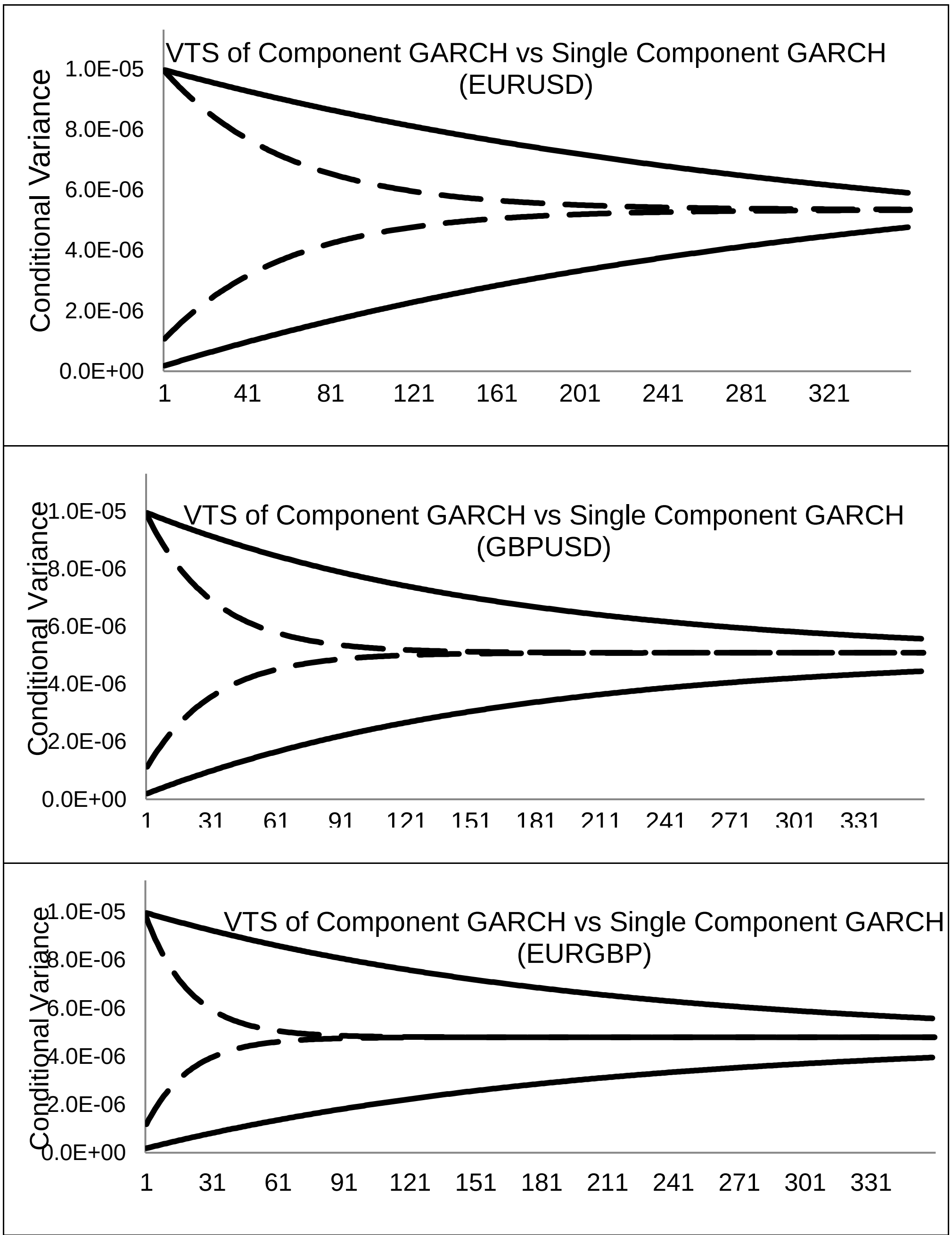
The volatility asymmetries are described by $\gamma s$ in AGH and by $\gamma_1$ and $\gamma_2$ in the ACGH model as this effect may have either transitory or permanent impacts. Their estimated values indicate negative return-volatility relation for all three exchange rates. The estimated $\gamma s$ of AGH model indicate that volatility asymmetry is statistically most significant for the GBPUSD exchange rate, followed by the GBPEUR and EURUSD exchange rate. By further examining this effect under the ACGH models, the volatility-asymmetry term is statistically significant in the transitory component for all exchange rates, but not at all for the long run volatility component. The short run asymmetry parameter $\gamma_1$ always has higher estimated values and t-statistic. In comparison, the long run asymmetry parameter $\gamma_2$ has estimated values about zero and are statistically insignificant.

Implicit reasons of the volatility asymmetry might be explained in light of the base currency effect. Higher expected GBPUSD volatility could lead British to sell USD denominated assets and Americans to sell GBP denominated asset. The same principal applies to the EURUSD exchange rate. However, to the extent that Euros have less global influence and scale of economic development than US dollars, such base-currency effect for Euros in exchange for the GBP is expected to be less significant than the GBPUSD and EURUSD exchange rates. In this regard, foreign exchange traders, particularly trading the Euros and British pounds in exchange for the US dollars, tend to be more responsive to any volatility asymmetry signals. In comparison, during the long time horizon it is expected that any exchange rate return and their volatility exhibit symmetric relations, since for any bilateral exchange rate the positive returns of one currency are always necessarily negative returns for the other (McKenzie 2002; Wang and Yang 2009).

Figure 4.2 plots the variance term structure (VTS) of component GARCH models (solid line) versus standard GARCH models (dash line) based on their volatility term structure functions (4.9) and (4.10). We plot both volatility term structures over 360 days, with the x-axis showing the time span and the y-axis showing the level of variance. In each figure, we plot the VTS with both high and low initial volatility, so as to exhibit the difference of two type models when converge to their long run unconditional volatilities. To facilitate the inspection, for VTS under all three currency pairs category, the low initial and high initial volatilities for both conventional and component GARCH models are set to be 1e-5 and 1e-6 respectively.

As all three constituent figures show, convergence to their long run unconditional variance is fastest for the single component GARGH models. In comparison, the convergence speed appears much slower for the component GARCH model, implying that the volatility evolution described by the component GARCH model is more persistent than the conventional GARCH models. Furthermore, by examining the convergence speed of the single component GARCH model it appears that its VTS under the EURGBP has the fastest convergence speed, while the speed under the USDEUR is slowest.

**Figure 4.2 GARCH volatility term structures**




Variance term structures of the component GARCH (solid line) against the single component GARCH (dashed line) using the parameters estimated from EURUSD, GBPUSD, and GBPEUR exchange rate returns (as reported in Table 4.4). All VTS are plotted for 1 through 360 days. For each figure, to exhibit the convergence speed we plotted both the high and low initial volatility of variance term structure. To facilitate the inspection, in the VTS of single component GARCH model the initial values for the high and low variance were set to be 1e-5 and 1e-6 respectively. For the component GARCH model, the initial high and low volatility was set to be 1e-5 and 1.4e-7, respectively.

### 4.5 The FX option valuation results

With GARCH parameters estimated, we evaluated the currency option pricing performance of GARCH models for the three specified exchange rates. To facilitate the assessment of the valuation results, we divided the option data into two categories subject to moneyness and time to expiration. Define the ratio of date t spot exchange rate to their exercise price as moneyness criteria, a call option is defined to be in-the-money if $X / K < 0.99$; near-the-money if $0.99 \le X / K < 1.01$; and out-of-the-money if $X / K \ge 1.01$. According to the time to expiration, the option contract is grouped to be short-maturity (< 30 days), medium-maturity (30-60 days), and long-maturity (> 60 days)[16].

With respect to the measurement of valuation errors, three loss functions in relation to the model price and market options were employed for our assessment. Let *n* denote contract number under any moneyness or maturity categories, and N the total number of contracts of the overall sample under each exchange rate category, $C_i$ and $C_i^M$ denote respectively the model option price and market option price, then we have three loss functions available for gauging the pricing results, which are the raw valuation errors (RVE), the absolute valuation error (AVE), and root mean absolute valuation error (RMVE),

[16] Previous studies applying the GARCH model in currency options evaluation with validated market option data can be found in Bollen and Rasiel (2003), Harikumar and Boyrie (2004), and Bhat and Arekar (2016). And our exercise has the filtration similar to Boller and Rasiel (2003) in which one year data with one-, two-, three-month data are considered only with the liquidity concern. In our study options with maturities more than three month appear incomplete frequently over certain days so we restrict the maturity to about 100 days only to have consistent comparative analysis.

$$RVE = \frac{1}{n}\sum_{1}^{n}\left(C_i - C_i^M\right),$$
$$AVE = \frac{1}{n}\sum_{1}^{n}\left|C_i - C_i^M\right|,$$
$$RMAE = \frac{1}{N}\sum_{1}^{N}\left|C_i - C_i^M\right|.$$

Table 4.5 summarises the foreign exchange option valuation results. Panel A reports the model's performance in the valuation of the EURUSD options. With respect to the model performance under the term to expiration categories, the Garman-Kohlhagen model outperform other models in pricing the short-maturity option while the component models underperform others with relatively large absolute valuation errors. The symmetric simple GARCH model and the asymmetric GARCH model have the best performance in valuing the medium term options. The component GARCH models, although producing better short term option pricing results, their pricing errors are greater than the conventional GARCH models. The apparent improvement of component GARCH model arises from the long-term option valuation. Both component GARCH models outperform other models. The asymmetric component model in particular yields the least valuation bias.

Concerning valuation performance under the moneyness categories, the GARCH models slightly underprice out-of-the-money options and the Garman-Kohlhagen model slightly undervalue in-the-money options. According to absolute valuation errors, the Garman-Kohlhagen model outperform others at pricing the in-the-money options. The asymmetric component GARCH model outperforms other models at pricing the at-the-money options. And the asymmetric GARCH model outperforms other models at the out-of-money option pricing. Overall, the asymmetric component model yields the least valuation errors, followed by the symmetric component model.

Panel B summarises the GBPUSD foreign exchange options valuation results. According to the term to expiration category, the GARCH models undervalue the short- and medium-term options, but overprice the long-term options. Under the moneyness categorised results, it seems all models have undervaluation of the in-the-money and at-the money options, implying potential systematic underestimating of GBPUSD volatilities. Unlike the USDEUR option pricing results, the Garman-Kohlhagen model yields the least absolute valuation errors across all categories, implying the success of the effectiveness of the deterministic volatility function. After the Garman and Kohlhagen model, it is the symmetric component model with the least valuation errors. For the standard GARCH model, unfortunately it underperforms all the remaining models.

The EURGBP foreign exchange option pricing results are summarised in Panel C. Under the maturity categories, the asymmetric GARCH model produces the least valuation errors for the short-term options. The standard GARCH model yields the least valuation errors for the medium-term options. And the asymmetric component GARCH model yields the least valuation

**Table 4.5 Valuation results of EURUSD, GBPUSD, and EURGBP options**

**Panel A. EURUSD Option pricing results**

| | G-K | | SGH | | AGH | | SCGH | | ACGH | |
|---|---|---|---|---|---|---|---|---|---|---|
| | RVE | AVE | RVE | AVE | RVE | AVE | RVE | AVE | RVE | AVE |
| Short Term | 0.022 | 0.124 | -0.069 | 0.189 | -0.068 | 0.194 | -0.092 | 0.214 | -0.092 | 0.215 |
| Medium Term | 0.098 | 0.250 | -0.002 | 0.100 | -0.004 | 0.105 | -0.067 | 0.141 | -0.072 | 0.135 |
| Long term | -0.060 | 0.143 | 0.079 | 0.246 | 0.134 | 0.206 | 0.025 | 0.138 | 0.013 | 0.128 |
| In-the-money | -0.020 | 0.111 | 0.047 | 0.158 | 0.061 | 0.150 | -0.026 | 0.120 | -0.032 | 0.116 |
| At-the-money | 0.137 | 0.330 | -0.009 | 0.204 | 0.014 | 0.188 | -0.033 | 0.212 | -0.040 | 0.201 |
| Out-of-the-Money | 0.032 | 0.189 | -0.065 | 0.178 | -0.049 | 0.173 | -0.090 | 0.196 | -0.093 | 0.193 |
| RMAE | 0.184 | | 0.175 | | 0.166 | | 0.164 | | 0.159 | |

**Panel B. GBPUSD option pricing results**

| | G-K | | SGH | | AGH | | SCGH | | ACGH | |
|---|---|---|---|---|---|---|---|---|---|---|
| | RVE | AVE | RVE | AVE | RVE | AVE | RVE | AVE | RVE | AVE |
| Short Term | 0.004 | 0.108 | -0.422 | 0.426 | -0.424 | 0.426 | -0.387 | 0.394 | -0.382 | 0.394 |
| Medium Term | 0.011 | 0.092 | -0.058 | 0.125 | -0.063 | 0.129 | -0.020 | 0.100 | -0.006 | 0.124 |
| Long term | -0.033 | 0.170 | 0.010 | 0.224 | 0.036 | 0.206 | 0.058 | 0.190 | 0.042 | 0.208 |
| In-the-money | -0.008 | 0.076 | -0.073 | 0.236 | -0.070 | 0.229 | -0.051 | 0.222 | -0.029 | 0.236 |
| At-the-Money | -0.027 | 0.142 | -0.129 | 0.267 | -0.121 | 0.259 | -0.091 | 0.238 | -0.079 | 0.252 |
| Out-of-the-Money | 0.010 | 0.181 | -0.236 | 0.246 | -0.237 | 0.251 | -0.192 | 0.213 | -0.211 | 0.227 |
| RMAE | 0.125 | | 0.260 | | 0.256 | | 0.230 | | 0.244 | |

| Panel C. GBPEUR option pricing results | G-K | | SGH | | AGH | | SCGH | | ACGH | |
|---|---|---|---|---|---|---|---|---|---|---|
| | RVE | AVE | RVE | AVE | RVE | AVE | RVE | AVE | RVE | AVE |
| Short Term | -0.091 | 0.279 | -0.130 | 0.151 | -0.130 | 0.146 | -0.156 | 0.179 | -0.156 | 0.176 |
| Medium Term | -0.027 | 0.253 | 0.068 | 0.225 | 0.068 | 0.230 | 0.001 | 0.273 | 0.009 | 0.270 |
| Long term | -0.091 | 0.345 | 0.149 | 0.200 | 0.153 | 0.203 | 0.032 | 0.122 | 0.037 | 0.116 |
| In-the-money | 0.067 | 0.240 | 0.124 | 0.150 | 0.126 | 0.151 | 0.014 | 0.120 | 0.018 | 0.116 |
| At-the-money | 0.083 | 0.378 | 0.015 | 0.174 | 0.022 | 0.179 | -0.022 | 0.203 | -0.014 | 0.197 |
| Out-of-the-money | 0.044 | 0.405 | -0.110 | 0.273 | -0.114 | 0.269 | -0.150 | 0.296 | -0.148 | 0.295 |
| RMAE | 0.295 | | 0.192 | | 0.193 | | 0.195 | | 0.191 | |

G-K, SGH, AGH, SCGH, ACGH refer to respectively the Garman and Kohlhagen model, the symmetric GARCH model, the asymmetric GARCH model, the symmetric component GARCH model, and the asymmetric component GARCH model. The moneyness criterion is defined by ratio of spot exchange rate to the exercise price, X/K, which include in-the-money group with X/K<0.99, near-the-money group with 0.99<= X/K <1.01, and out-of-the-money group that X/K>=1.01. Short-, Medium-, and Long maturity are respectively the categories with option maturity that within 30 days, between 30 and 60 days, and option maturities longer than 60 days. RVE, AVE denote the raw valuation error and absolute error, respectively. RVE takes the price differential between model option price and market option price. AVE is the RVE in absolute value. Root mean absolute errors are the average value of overall absolute valuation errors.

results for the long-term options, which is in line with its performance in the previous two currency options valuation. In the aspect of moneyness categories, the component GARCH model outperform remaining models in in-the-money option evaluation, and single component GARCH model outperform other models in the at-the-money and out-of-the money option evaluation. By and large, the component GARCH models produce valuation error at about the same magnitude with the asymmetric component model produces relatively less pricing errors. The second model with relatively less valuation errors is the standard GARCH model.

**Table 4.6 Overall valuation results**

| | Short-term | Medium-term | Long-term |
|---|---|---|---|
| ITM | G-K | SGH | SCGH |
| | ACGH | AGH | ACGH |
| NTM | G-K | SCGH | ACGH |
| | ACGH | SGH | SCGH |
| OTM | G-K | SGH | SCGH |
| | SCGH | SCGH | ACGH |

Table 4.6 further aggregates all aforementioned valuation results in order to have more general conclusion of model performance, irrespective of any specific exchange rates. Although the results seem relatively mixed, we still could obtain some consistent findings across moneyness and maturity groups. For instance, within the short maturity interestingly the Garman-Kohlhagen model and the component GARCH model perform relatively best. But in valuing the long term options it is always the component GARCH model outperforming the remaining models, which suggests that the effectiveness of component framework for remedying the deficiency of conventional GARCH models in long term option valuation.

Finally, we test the significance of valuation results with the findings reported in Table 4.7. The Wilcoxon signed ranks test was applied and tested the significance of the relative magnitude and difference of the valuation results (Krishnamoorthy 2016). To begin with, we ranked all values between the model option price and the market option price in order of their absolute size. We then affixed the sign to the difference score to each rank, in order to indicate later which rank resulted from positive difference scores and which rank resulted from negative difference scores. Let $T^+$ denote the ranks sum from the positive difference scores and N denotes the sample size. We can have the test statistics

$$z = \frac{T^+ - N(N+1)/4}{\sqrt{N(N+1)(2N+1)/24}},$$

which has approximately the normal distribution with zero mean and unit variance. In case that the sum of the positive ranks $T^+$ does not statistically different from the sum of the negative ranks $T^-$, we accept that the null hypothesis $H_0$ that the treatment of model price and the treatment of market option price are equivalent. This suggest both samples are from population with same median and continuous distribution. Our alternative hypothesis is that the model price and market price have different medians and distribution, i.e., the sum of the positive ranks differs from the sum of the negative ranks. It should be noted that the Wilcoxon signed ranks test gives more weight to the large pricing bias between the model price and market price and gives less weight to valuation errors with small magnitude.

**Table 4.7 The Wilcoxon signed ranks test results**

| | | G-K | SGH | AGH | SCGH | ACGH |
|---|---|---|---|---|---|---|
| EURUSD | $T^{+}$ | 20136 | 17451 | 16026 | 20716 | 21394 |
| | *p*-value | 0.3858 | 0.8087 | 0.2011 | 0.01845 | 3.78e-3 |
| | conclusion | retain $H_0$ | retain $H_0$ | retain $H_0$ | Accept $H_1$ | Accept $H_1$ |
| GBPUSD | $T^{+}$ | 25782 | 31640 | 31855 | 28289 | 28212 |
| | *p*-value | 0.3319 | 6.22E-15 | 1.83E-15 | 6.02E-8 | 8.17E-8 |
| | conclusion | retain $H_0$ | Accept $H_1$ | Accept $H_1$ | Accept $H_1$ | Accept $H_1$ |
| EURGBP | $T^{+}$ | 27632 | 11435 | 11565 | 14790 | 14344 |
| | *p*-value | 1.74E-3 | 0.1914 | 0.2406 | 0.034 | 0.095 |
| | conclusion | Accept $H_1$ | retain $H_0$ | retain $H_0$ | Accept $H_1$ | retain $H_0$ |

Two related samples, the option price calculated from models and the recorded price from the traded options in market, are used for the Wilcoxon signed rank test. The testing procedure utilises the difference score, i.e., the raw valuation errors, which are ranked in order of absolute magnitude. The null hypothesis is that the model option price and the market option price do not differ from others. And the alternative hypothesis is that the model prices differ from market option prices, in the sample mean and continuous distribution aspects.

Table 4.7 reports the testing results of the raw valuation errors (RVE) summarised in Table 4.5. In the pricing of the EURUSD options, the valuation results from the Garman-Kohlhagen model, the standard GARCH model and the asymmetric GARCH model have the same median and continuous distribution as the market option sample. But this is not the case for the component models, although they yield less valuation errors. For the GBPUSD option pricing results, surprisingly the option price from all of the GARCH models are statistically distinctive from the market option price. In comparison, the Garman and Kohlhagen model results retain the null hypothesis. The implicit reasons would be that the GARCH model price has the systematic underestimation of the GBPUSD exchange rate volatilities. Lastly, for the EURGBP test results the GARCH models appear to have the optimistic performance. Except for the symmetric component GARCH other GARCH models all produce a theoretical option price that has no difference to the market option price. For the Garman and Kohlhagen model, it produces option values that differ from market price with distinct median and distribution.

### 4.6 Conclusion

In this study we have examined the performance of component GARCH models in modelling EURUSD, GBPUSD, and the GBPEUR exchange rate volatilities and valuing the foreign exchange options written on these currencies. The results show that the component GARCH model always fit the exchange rate better than the single volatility component GARCH model. Our results complement previous studies that currency volatility is symmetric in nature in responding to new information, but that the symmetry mainly exists in the long run. For short time horizons, there is significant volatility asymmetry. Further, the estimated GARCH model with long run volatility component yields an improved performance in the simulated option valuation, particularly in the valuation of the currency options with long term to expiration.

Given existing research on GARCH models for pricing currency options, our primary contribution attribute to the complementary of existing literature by using the component GARCH model for currency option valuation. As our empirical assessment illustrates, by factoring the transitory and permanent components in the conventional GARCH model, the extra control is obtainable in simulating the option price over the longer time horizon. However, our empirical investigation has the limitation of assuming a normal distribution, which may not be accurate for all situations, so it would be interesting to assess the applicability non-Gaussian distributions in describing the volatility dynamic for exchange rates, its impact on the precision of the simulated option price and its significance for hedging.

# Chapter 5

## Evaluating the price information and information uncertainties of canola spot and futures markets in Canada[☆]


### Abstract

The canola futures market has the largest trading volume among other commodities in Canada. This chapter aims to analyse the interrelation between its spot and futures price. A cointegration procedure with the threshold test confirms the price equilibrium between the two markets with a no-arbitrage band, and its futures market price adjusts more actively to any disequilibrium. This inter-market stylised fact is further revealed by a bivariate GARCH volatility analysis, as empirical results indicate that the effect from volatility clustering runs from the futures market to its spot market.



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## 5.1 Introduction

Canola futures amounts to the largest volume commodity futures in Canada with assurance of market depth and trading liquidity. In Canada, the canola spot market is subject to the least government regulation and subsidy, and is virtually free from the institutional factors and policy considerations that interfere with market behaviour and create pricing distortions (Kaastra 2014). However, given the importance of this commodity and its futures market, research on it is unfortunately sparse in comparison with other heavily traded commodities such as crude oil, wheat, coffee, and metals. In this chapter we aim to study this primary commodity in Canada with respect to its price movement and price volatility between its futures and spot markets.

Conventional studies on the spot and futures price relationship claim that a price equilibrium could arise because the futures market reflect the market expectation of the spot price. This implies that the price behaviour for the two markets is described by a long run co-movement process with a mean-reverting price differential, referred to as a cointegration relationship. The subsequent studies of Balke and Fomby (1997) and Hansen and Seo (2002) further specify the price adjustment system maintaining the price equilibrium by incorporating a threshold in the error correction mechanism, since arbitrage is only permitted to step in when the profit accruing from the process exceeds the potential cost from transaction and any other adjustments, thereby creating a no-arbitrage band. The spot-futures price behaviour is thus represented by a threshold error correction model, in which the price behaviours and interactions under each regime are separately specified. Finally, because the cointegration and error correction model reveal information on the price level and change for each market, any additional information on uncertainty and particularly volatility runs the risk of being neglected. We aim to integrate this implicit aspect about information so as to conduct a more explicit pattern of the price information pattern and character for the canola spot-futures price relationship.

We adopted a statistical methodology based on a cointegration and a threshold error correction model with a bivariate-GARCH model for analysing the canola spot-futures price information analysis, and set four research objectives for our investigation. These are (i) to identify the existence of a dynamic price equilibrium between the spot and futures price for canola in Canada; (ii) to test the presence of a threshold effect in addition to a cointegration relationship; (iii) to use these results to enquiry the adjustment process between the two markets; and (iv) to investigate the volatility dynamic of the canola spot and futures markets. We expect that results from our aggregate analysis will provide more explicit information about how canola spot and futures price movement affect each other, and uncertainties of both spot and futures price movements.

By examining the threshold cointegration over the canola spot futures market, our investigation contributes to the literature on this important commodity market under a non-linear empirical methodology, as well as characterising the co-movements of the spot and futures prices. Conventional cointegrations claims that the adjustment from either the spot or futures market to their equilibrium deviations occurs instantaneously at each period. However, in the presence of transaction cost or market conditions caused structural change, there could exist inactions and threshold over the price adjustment process.

Hence the economic forces such as arbitrage only steps in when profits exceed certain expenses such as transaction cost. Therefore, in our methodology we adopt a threshold model accommodating this non-linearity and meanwhile allowing for regime-switching during the error correction process which maintains the spot futures price equilibrium. Our results show that threshold exists between the canola spot-futures market and the presence of a no-arbitrage band. Also, the adjustment maintaining the price equilibrium takes place in the futures market and these adjustments take place only when the spot and futures deviation exceeds this band.

Furthermore, with a bivariate GARCH model of BEKK class, the results show that both canola spot and futures market price exhibit strong volatility persistence. The volatility spillover impact runs from the futures market to the spot market at a subsequent time point, implying that it is futures trading that destabilizes the volatilities in the spot market. Finally, unlike previous findings arguing for a positive return-volatility association, in our assessment no evidence is found for any volatility asymmetry, which potentially emphasizes the market depth of canola futures market in Canada and its symmetric return-volatility relationship.

The rest of this chapter is organised as follows. Section 5.2 surveys the related literatures of canola futures and identify the limitations and deficiencies of previous researches. Section 5.3 discusses the data details and conducts descriptive statistics for the data sample, which is an important preliminary analysis prior to the testing of the cointegration and price information analysis. Section 5.4 presents cointegration analysis and test results. In Section 5.5, the threshold effect is examined and an error correction model with threshold is estimated. The price adjustment of the spot (futures) market toward their equilibrium is examined in this section. In section 5.6, a bivariate GARCH model of BEKK class is employed to examine the volatility behaviours of each market and their interactive volatility dynamic. As threshold cointegration and error correction models focus on the price level and price return of the spot and futures market interrelation, the volatility analysis is our subsequent methodology which focuses on the second moment for examining the information uncertainty. Summary and concluding remarks are provided in section 5.7.

## 5.2 Related studies

Empirical studies on the price behaviour of canola market can be found in Khoury and Yourougou (1991), Sephton (1992), Sephton (1993), Brockman and Tse (1995), Carter (1996), and Adämmer et al (2016) to date. Khoury and Yourougou (1991) and Brockman and Tse (1995) investigate the interrelationship between the canola spot-futures market price and conclude that traders with market-wide information prefer to enter the futures market first. They further demonstrated that the existence of a canola futures market leads to an improvement in pricing efficiency for the canola spot market and that the futures price can be represented by a forecasted cash price plus an expected risk premium. In contrast, Carter (1996) addresses the issue of rail transport regulation by studying its impact on the canola spot-futures market relationship, and concludes that the canola spot and futures price do not converge over the futures delivery month, which suggests inefficiencies in the price discovery process for the futures market.

Sephton (1992) examines the extent to which the macroeconomic events such as inflation and currency depreciation affect the commodity markets of barley, canola and wheat. Results show that currency depreciation affects commodity price in the short run. But, in the long run there is no lasting effect. In a further study on the Winnipeg commodity exchange futures, Sephton (1993) estimates the volatility of these commodities and suggests that the GARCH model based results provide a superior hedge ratio. Adämmer et al (2016) assessed the price transmission and spillover dynamic of canola, corn and wheat futures between north American and European markets. They found that the U.S. futures price of these commodities play the leading role in price transmission and predominantly react to the deviation from the long run equilibrium with the European agricultural markets.

The aim of the current study is to examine the price relationship between the canola spot and futures market but under a non-linear threshold framework. An important assumption underpinning cointegration and the error correction model is linearity. In Sephton (1992), Brockman and Tse (1995), and Adämmer et al (2016), the linear approaches of Engle and Granger (1987) procedure and the Johansen procedure are used for verifying the existence of a long run price equilibrium. An implicit shortcoming of their linear cointegration approach is the inadequate measurement over the possible structural change during the constant spot-futures price equilibrium over time. However, the presence of transaction costs makes a time-invariant price relationship over the long run to be somewhat unlikely. Balke and Fomby (1997) and Wang and Wu (2013) argue that an effective arbitrage process moderating the long term price equilibrium can only arise provided the price disequilibrium significantly exceeds the combined transaction and adjustment costs. In essence, there exists a no-arbitrage band within which arbitrage is deferred until some threshold when the combined transaction and adjustment costs are fully compensated. This threshold has the effect of introducing a potential non-linearity in the representation and creating a structural change issue, which needs to be addressed for the investigation to provide robust findings.

### 5.3 Data properties and preliminary analysis

The empirical analysis of the current study was performed using the canola futures price from the ICE (Intercontinental Exchange) and the cash price from the ACPC (Alberta Canola Producers Commission). Time series for both canola spot and futures price were selected to span from January 2003 to April

**Table 5.1 ICE canola futures average trading volume**

| | Front-month | Second-nearest | Third-nearest | Fourth-month |
|---|---|---|---|---|
| 2003 | 2804 | 2052 | 1479 | 238 |
| 2004 | 2277 | 2189 | 402.2 | 83 |
| 2005 | 2872 | 1862 | 661 | 132 |
| 2006 | 5096 | 3338 | 526 | 104 |
| 2007 | 5489 | 4156 | 604 | 299 |
| 2008 | 6174 | 4603 | 949 | 516 |
| 2009 | 6609 | 3427 | 419 | 138 |
| 2010 | 8011 | 4918 | 761 | 245 |
| 2011 | 9404 | 5599 | 1008 | 534 |
| 2012 | 8119 | 6524 | 2130 | 761 |
| 2013 | 9395 | 5891 | 1680 | 683 |
| 2014 | 7844 | 7259 | 2489 | 1066 |
| 2015 | 9607 | 7889 | 2084 | 1032 |

The yearly average trading volume summary of the ICE canola futures contract. The front month contract refers to the contract with the closest settlement date. The second-nearest month contract refers to the futures contract with the settlement date right after the front month contract, and similar routines apply to the third and fourth nearest month contract. All trading volume data is obtained from Thomson Reuters Datastream.

2016 with 3246 daily observations[17]. The nearby futures contract with the closest settlement date was used to construct the futures price series because of being the most actively traded. This is exhibited in Table 5.1, which shows the yearly trading volume from the nearby to the fourth distant futures price series. Note that the volume of trading increasing dramatically for both the front-month and second-nearest month contracts, suggesting the strong growth of canola futures trading in futures during the sample period.

Descriptive statistics for the canola spot and futures prices and their returns (based on the first difference in logarithmic form) are summarised in Table 5.2. As can be seen, the spot canola price has a greater mean, which implies backwardation that reflects market expectations for the futures price. It also has a greater standard deviation with a greater maximum and lesser minimum. Under the first difference category, the returns of spot and futures series are about zero, positively skewed and heavy-tailed, implying frequent small losses but few extreme gains. Jarque-Bera test results reject the null hypothesis of normal distribution for all series (Brooks 2008). The Lagrange multiplier statistics indicate the presence of a significant ARCH effect, a phenomenon which is consistent with the observed excess kurtosis. The Ljung and Box (1978) statistics, by examining the autocorrelations of each price series up to five, ten, and fifteen lags, reject the null hypothesis for all four-time series that data are independently distributed and have serial correlations.

[17] When enquiring the historical canola cash price from Alberta canola production commission, the longest times series of daily canola price they can provide is starting from January 2003.

**Table 5.2 Sample descriptive statistics**

| | Price level | | First difference | |
|---|---|---|---|---|
| | Spot prices | Front Month futures | Spot Price | Front Month futures |
| Observations | 3246 | 3246 | 3245 | 3245 |
| Mean | 465.56 | 444.60 | -4E-05 | -4E-05 |
| Std.dev | 114.72 | 107.61 | 0.014 | 0.014 |
| Min | 245.70 | 237.50 | -0.10 | -0.14 |
| Max | 769.00 | 759.00 | 0.11 | 0.14 |
| Skewness | 0.103 | 0.149 | 0.53 | 0.39 |
| Kurtosis | -0.808 | -0.721 | 7.43 | 10.42 |
| JB test | 93.96 | 82.32 | 7.62E+03 | 1.47E+04 |
| Q (5) | — | — | 16.26 | 21.21 |
| Q (10) | — | — | 29.09 | 31.71 |
| Q (15) | — | — | 47.72 | 46.15 |
| LM test (12) | — | — | 1033 | 2076 |
| ADF | -0.58 | -0.58 | -42.48 | -41.69 |
| PP | -5.30 | -5.81 | -3076.37 | -2937.16 |
| KPSS | 16.85 | 14.43 | 0.10 | 0.09 |

ICE canola futures and canola spot price are in logarithm with the first difference of the log prices. Std.dev is the standard deviation of each time series. The JB test is the Jarque-Bera normality test. The Q (5), Q(10), and Q(15) refer to the Ljung-Box test with 5, 10 and 15 lags respectively. LM test refers to the Lagrange Multiplier test for the heteroscadsticity as in Engle (1982). The ADF, PP, and KPSS test are unit root tests, and are abbreviations of the augmented Dickey-Fuller test, Phillips-Perron test, and the Kwaitkowski-Phillips-Schmidt-Shin test, respectively.

The stationarity of canola spot and futures price are examined using the augmented Dickey-Fuller (ADF) test, the Phillips-Perron (PP) test, and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests. The ADF test and PP test are customary in previous work. The Phillips-Perron test has the advantage over the Dickey-Fuller test due to its non-parametric feature, irrespective of pre-selected serial correlation level. However, both tests have low testing power against stationary series with a root near non-stationary boundary (Brooks 2008). For complementary, we employed the KPSS test to further verify the stationarity. In all cases, test results show that both spot and futures price series are non-stationary and integrated of order one, but their first difference is I(0) which is stationary, regardless of which test is used.

### 5.4 Testing for Cointegration

The canola spot and futures price should normally be in equilibrium because they share a similar information set. Any significant divergence creates a profitable arbitrage opportunity, which stimulates buying and selling pressures that result in eliminating the price differential. In the context of econometrics, this phenomenon refers to a cointegration relationship in which the two

**Figure 5.1 Historical price of canola spot and futures (2003-2016)**

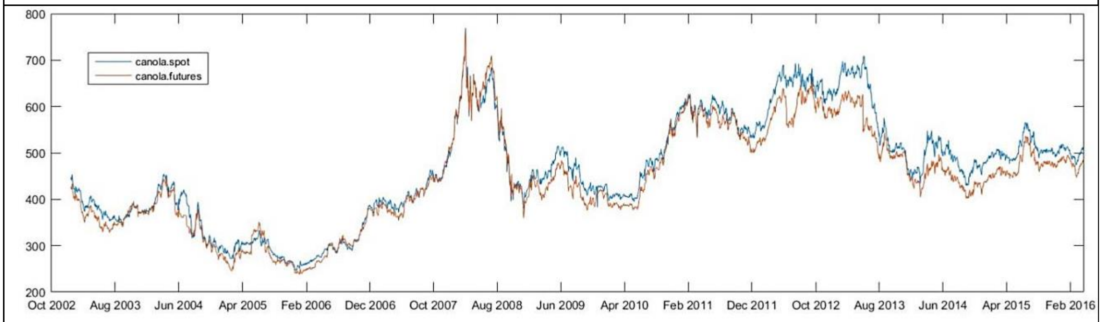
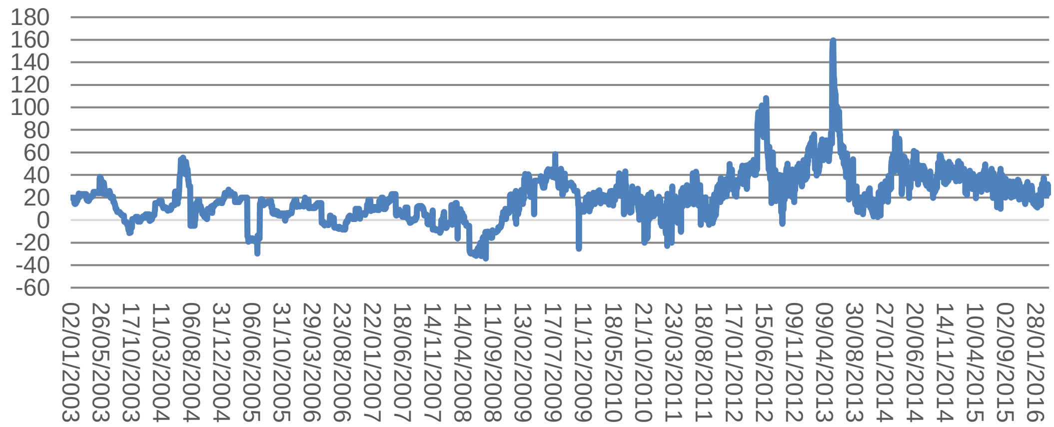



**Figure 5.2 Price spread between the canola spot and futures price**




economic variables could drift away during the short run but an economic force such as arbitrage in the long run will bring them back as their disparity exceeds some thresholds. Figure 5.1 presents the canola spot and the futures series at price level, and their spread in Figure 5.2. As both figures show, the two series appear as a random walk and non-stationary, but have similar trend binding them together over the long term. Although over certain periods the two series drift apart, their spread remains mean-reverting over long term.

The procedure of Johansen (1988) and Johansen and Juselius (1990) was used to test for cointegration. Although previous models of Engle and Granger (1987) and Phillips and Ouliaris (1990) also provide a cointegration framework, their works have an inherent issue in that during the estimation of the interchange of dependent and independent variables could result in estimating bias (Brooks, 2008). The Johansen procedure overcomes this problem by providing a symmetric treatment to any change of dependent variables within the multivariate system. To illustrate the procedure, let $X_t$ be a *p*th order autoregressive process, $[x_{1t}, x_{2t}, \ldots, x_{nt}]'$, $\Delta$ be the first difference operator, the Johansen cointegration can be written as,

$$\Delta X_t = \alpha + \Theta X_{t-1} + \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_n \Delta X_{t-n} + e_t. \tag{5.1}$$

In (5.1), $\Delta X_t = X_t - X_{t-1}$, and each variable in $\{X_1, \ldots, X_n\}$ is integrated of order one, i.e. contain one unit root. Θ is a $n \times n$ parameter matrix with rank of the number of independent cointegrating vectors. For cointegration to be satisfied, the right hand side of (5.1) must represent a stationary process, implying that $\Theta X_{t-1}$ is stationary. Thus to test the cointegration in the multivariate system (5.1) we examine the rank of the Θ in (5.1), which equals the number of characteristic roots (eigenvalues) which have a non-zero value. Johansen and

Juselius (1990) propose a trace test for examining the number of non-zero eigenvalues in Θ. Assuming there are *M* eigenvalues in the system, then the null hypothesis $H_0$ and alternative hypothesis $H_1$ for the trace test are

$$H_0 : m \leq M \text{ vs } H_1 : m > M \tag{5.2}$$

If T denotes the sample size, *n* the number of integrated variables in the system, while the eigenvalues of $\Theta$ are real numbers such that $1 > \phi_i > \ldots \geq 0$, we test the $\lambda_{trace}$ which is given as $\lambda_{trace}(r) = -T\sum_{i=R+1}^{n}\ln(1-\phi_i)$ and $\lambda_{\max}(r, r+1) = -T\ln(1-\phi_{i+1})$. The statistics $\lambda_{trace}$ follows a nonstandard distribution and tests the null hypothesis that there is less than or at most *r* cointegration vectors against its alternative hypothesis of more than *r* cointegration vectors existed within the system. Critical values of $\lambda_{trace}$ statistics are provided in Johansen and Juselius (1990). In our study, $n = 2$, since only the spot and the nearby futures price series are used.

Table 5.3 reports the Johansen test results between the canola spot and futures price series. A cointegration model under vector autoregressive structure with two lags is determined using the Schwarz Bayesian Criterion in its multivariate version (Schwarz 1978). The trace statistic tests the null hypothesis that the given system has at most *r* cointegrating vectors against its general alternative hypothesis, while the max-eigenvalue statistic tests the null that the system exhibits *r* cointegrating vectors against the alternative hypothesis of r+1 cointegrating vectors. The results in panel A show that the trace test statistic is 40.54 under the null hypothesis $r = 0$, which is higher than

**Table 5.3. Testing cointegration between spot and nearby futures price**

| Panel A. Johansen procedure using trace test | | | | |
|---|---|---|---|---|
| | Trace Statistics | 10 percent Critical Value | 5 percent Critical Value | 1 percent Critical Value |
| r≤1 | 3.06 | 7.52 | 9.24 | 12.97 |
| r=0 | 40.54 | 17.85 | 19.96 | 24.6 |
| Panel B. Johansen procedure using Max-eigen value test | | | | |
| | Max-eigen Statistics | 10 percent Critical Value | 5 percent Critical Value | 1 percent Critical Value |
| r≤1 | 3.06 | 7.52 | 9.24 | 12.97 |
| r=0 | 37.48 | 13.75 | 15.67 | 20.20 |

Note: The critical values are obtained from Johansen and Juselius (1990). The trace test and max-eigen value test reveal the cointegration at the 0.01 level.

the 1 per cent critical value of 24.6 and reject the null hypothesis of zero cointegrating vector existed in the system. In panel B, we reached the same conclusion since the max-eigen value statistic of 37.48 is greater than the highest critical value of 20.2, which also rejects the null hypothesis $r = 0$ at the 1% level. Furthermore, under both approaches using trace test and max-eigenvalue tests, the test statistic value of 3.06 has a smaller value than all critical values under the null hypothesis r≤1. In conjunction with previous conclusions that the system has at least one cointegrating vector, we confirm that only one cointegration relationship exists within the system.

### 5.5 Testing the threshold and the threshold error correction model

Under the linear framework, an implicit assumption underpinning both the cointegration and error correction models is that any deviation from equilibrium leading to an adjustment is instantaneously corrected. Moreover, the magnitude of this adjustment bears no relation to the size of the deviation. In contrast, Hansen and Seo (2002) and Hwan Seo (2010) argue against this being a complete description and respond by suggesting that for a price adjustment to occur, the price deviation has to exceed a no-arbitrage band so that the benefits accruing from arbitrage are significantly greater than the transaction cost.

**Table 5.4 Hansen and Seo test of the threshold cointegration**

| | | | | |
|---|---|---|---|---|
| Test statistic: | 50.57 | (Maximized for threshold value: -25.2) | | |
| P-value: | 0 | (Fixed regressor bootstrap) | | |
| Critical Values: | 0.90% | 0.95% | 0.99% | |
| | 25.47 | 27.55 | 31.8 | |

Notes: The Hansen and Seo (2002) sup-LM test was used for testing the null hypothesis of linear cointegration against the alternative hypothesis of threshold cointegration. Test statistics were obtained using the parametric bootstrap procedure with 5000 simulations and a 300 grid point search.

We performed the sup-LM test of Hansen and Seo (2002) to assess the evidence of threshold cointegration. Providing the estimated cointegration value, the LM (Lagrange multiplier) test was implemented over a range of different threshold values, then the threshold test reports the identified threshold value which maximizes the LM test results. Table 5.4 reports the test results, where the test statistics were obtained using 5000 simulation replications of the bootstrap procedure and 300 grid points search[18]. The sup-LM test statistic of 50.57 is greater than the 0.99% critical level so we rejected the null hypothesis of linear cointegration. The estimated threshold, -25.2, indicates the range of no-arbitrage band, implying that the arbitrage activity will be triggered when spot-futures price disparity exceeds this level.

[18] We set simulation times and grid points searching amount in line with Hansen and Seo (2002).

### 5.5.1 The threshold error correction model

Having demonstrated the long run cointegrating relationship between canola spot-futures price and the presence of the threshold over cointegration, it proceeded to the ECM (error correction model) analysis of the canola spot-futures price in the short run and to specify in what mechanism the spot and futures adjust from their disequilibrium values. The following threshold error correction model is estimated with two regimes:

$$\left.\begin{aligned} \Delta S_t^{(1)} &= \omega_{11}^{(1)} + \theta_1^{(1)}(\gamma_1 S_{t-1} + \gamma_2 f_{t-1}) + \sum\nolimits_{i=1}^{p} \alpha_{1,i}^{(1)} \Delta S_{t-i} + \sum\nolimits_{j=1}^{q} \beta_{1,j}^{(1)} \Delta f_{t-i} + \varepsilon_{1,t-1} \\ \Delta f_t^{(1)} &= \omega_{21}^{(1)} + \theta_2^{(1)}(\gamma_1 S_{t-1} + \gamma_2 f_{t-1}) + \sum\nolimits_{i=1}^{p} \alpha_{2,i}^{(1)} \Delta S_{t-i} + \sum\nolimits_{j=1}^{q} \beta_{2,j}^{(1)} \Delta f_{t-i} + \varepsilon_{2,t-1} \end{aligned}\right\} \quad (5.3a)$$

$$\left.\begin{aligned} \Delta S_t^{(2)} &= \omega_{11}^{(2)} + \theta_1^{(2)}(\gamma_1 S_{t-1} + \gamma_2 f_{t-1}) + \sum\nolimits_{i=1}^{p} \alpha_{1,i}^{(2)} \Delta S_{t-i} + \sum\nolimits_{j=1}^{q} \beta_{1,j}^{(2)} \Delta f_{t-i} + \varepsilon_{1,t-1} \\ \Delta f_t^{(2)} &= \omega_{21}^{(2)} + \theta_2^{(2)}(\gamma_1 S_{t-1} + \gamma_2 f_{t-1}) + \sum\nolimits_{i=1}^{p} \alpha_{2,i}^{(2)} \Delta S_{t-i} + \sum\nolimits_{j=1}^{q} \beta_{2,j}^{(2)} \Delta f_{t-i} + \varepsilon_{2,t-1} \end{aligned}\right\} \quad (5.3b)$$

In (5.3a) and (5.3b), $S_t^{(i)}$ and $f_t^{(i)}$ denote the spot and futures price at time *t*. $\alpha$s and $\beta$s are coefficients measuring the impact from lagged spot and futures price change. $\gamma_1 S_{t-1} + \gamma_2 f_{t-1}$ is the ECT (error correction term) at time t-1 with $[\gamma_1,\ \gamma_2]$ the cointegrating vector describing how spot-futures price disparity are corrected in the next period. Expressions with superscript "(1)"and "(2)" respectively denote the Regime 1 with $ECT_{t-1} < C$ and the Regime 2 with $ECT_{t-1} \geq C$ ,where *C* refers to the threshold level of no-arbitrage-band that triggers an arbitrage (Sephton 2003; Theissen 2012).

It should be emphasized that the coefficients under the threshold error correction model except the ECT will have different values within regime (1) and (2). Therefore, under the threshold framework the regime transition does not occur gradually and, instead, change only abruptly. As per Balke and

Fomby (1997), abrupt regime switching is a more realistic description since arbitrageurs must trade quickly to take profitable opportunities. Therefore the further the price disparity is away from their equilibrium the stronger is the arbitrage force bringing disparity back to the equilibrium.

The loading parameter $\theta$s determine the speed and direction of price adjustment. We expect that $\theta_1^{(1)}$ and $\theta_2^{(1)}$ to differ from $\theta_1^{(2)}$ and $\theta_2^{(2)}$ in both sign and value, because the price adjustments are treated distinctly for each regime. To evaluate the extent to which the spot and futures market attribute to adjustment, the common factor weights $CFW_s^{(i)}$ and $CFW_f^{(i)}$ are calculated to measure the relative contribution from the spot and futures market in price discovery. Note the Regime 2 is considered only because it is where the state price adjustment occurs.

$$CFW_s = \frac{|\theta_1^{(2)}|}{|\theta_1^{(2)}| + |\theta_2^{(2)}|} \quad \text{and} \quad CFW_f = 1 - CFW_s = \frac{|\theta_2^{(2)}|}{|\theta_1^{(2)}| + |\theta_2^{(2)}|} \tag{5.4}$$

The denominator in (5.4) represents the aggregate adjustment from both spot and futures price to any departure from their price equilibrium. The values of $CFW_S$ and $CFW_f$ are confined to [0, 1].

Following Schwarz and Szakmary (1994), Jesus Gonzalo (1995), Theissen (2012) and Admmer et al. (2015), if $CFW_f$ =1, then the price discovery process is driven by the futures market and the restoration of price equilibrium falls solely on the spot market; If $CFW_f = 0$, price discovery is driven by the spot market and the restoration of price equilibrium falls solely on the futures market;

and if $CFW_s = CFW_f = \frac{1}{2}$, each market contributes equally to the price discovery process.

Table 5.5 reports the estimated results of the TVECM (Threshold vector error correcting model) of (5.3a) and (5.3b), with the conditional least squares approach used as a model estimator[19]. In line with optimal lag length determined in cointegration, the two-lag length is retained for error correction model. Also with the identified threshold -25.2 and cointegrating vector $[1, -1.05]$, two regimes are identified. These are Regime 1 specified by $S_{t-1} - 1.05 f_{t-1} < -25.2$ with 8.8% of the observations, and Regime 2 specified by $S_{t-1} - 1.05 f_{t-1} \geq -25.2$ with 91.2% of the observations. While the left-hand side of the inequality refers to the price deviation between the spot and futures price, the right hand side refers to the level of no-arbitrage band. The results indicate an active error correction mechanism over this regime exists for 91.2 percent of whole sample, where the spot-futures price deviation exceeds the no-arbitrage band.

[19] For detailed formulation for the model estimator see Hansen and Seo (2002). The R 3.3.0 **tsdyn** package with **TVECM** function is used to identify the threshold effect and estimate the threshold error correction models.

## Table 5.5 TVECM estimating results

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Regime 1 ($\mathrm{ECT}_{t\text{-}1} \leq -25.2,$ , 8.8% of observations) | | | | | | | | |
| | $\omega_{1,1}^{(1)}$ | $\theta_{1}^{(1)}$ | $\gamma_1$ | $\gamma_2$ - | $\alpha_{1,1}^{(1)}$ | $\alpha_{1,2}^{(1)}$ | $\beta_{1,1}^{(1)}$ | $\beta_{1,2}^{(1)}$ |
| $\Delta S$ | -0.34 | -0.03 | 1 | 1.05 | -0.58*** | 0.04 | 0.59*** | -0.12 |
| | (0.79) | 0.35 | | | (2 .2E-27) | (0.54) | (5E-28) | (0.05) |
| | $\omega_{2,1}^{(1)}$ | $\theta_{2}^{(1)}$ | $\gamma_1$ | $\gamma_2$ | $\alpha_{2,1}^{(1)}$ | $\alpha_{2,2}^{(1)}$ | $\beta_{2,1}^{(1)}$ | $\beta_{2,2}^{(1)}$ |
| $\Delta f$ | -2.26 | -0.06 | 1 | 1.05 | 0.02 | 0.22*** | -0.019 | -0.34*** |
| | (0.10) | (0.11) | | | (0.72) | (0.5e-3) | (0.74) | (5.9e-7) |
| Regime 2 $(ECT_{t-1} > -25.2,$ 91.2% of observations) | | | | | | | | |
| | $\omega_{1,1}^{(2)}$ | $\theta_{1}^{(2)}$ | $\gamma_1$ | $\gamma_2$ | $\alpha_{1,1}^{(2)}$ | $\alpha_{1,2}^{(2)}$ | $\beta_{1,1}^{(2)}$ | $\beta_{1,2}^{(2)}$ |
| $\Delta S$ | -0.05 | -0.01 | 1 | -1.05 | -0.23*** | -0.11*** | 0.62*** | 0.12*** |
| | (0.68) | (0.11) | | | (1.6E-21) | (2.1E-7) | (5.8e-157) | (1.1e-6) |
| | $\omega_{2,1}^{(2)}$ | $\theta_{2}^{(2)}$ | $\gamma_1$ | $\gamma_2$ | $\alpha_{2,1}^{(2)}$ | $\alpha_{2,2}^{(2)}$ | $\beta_{2,1}^{(2)}$ | $\beta_{2,2}^{(2)}$ |
| $\Delta f$ | -0.04 | 0.02* | 1 | -1.05 | 0.04 | 0.02 | 8.70E-02*** | -0.07*** |
| | (0.75) | (0.02) | | | (0.15) | (0.33) | (2e-4) | (8.8e-3) |

Note: The threshold error correction model estimating results using canola spot and nearby futures price series. Two regimes with threshold value of -25.2 are found in model estimation. The values are coefficients of independent variables corresponding to equations set (3). Standard errors for each parameter are presented in parenthesis. ***, **, and * respectively denote 0.1%, 1%, and 5% significant level. The lag-length is determined by minimising the Bayesian information criterion. The threshold is identified by the search on grids of potential value. The error correction model parameters are estimated based on the Conditional Least Squares Approach.

The estimated $\beta_{1,j}$ and $\alpha_{2,i}$ reveal the short run predictive role of a futures price change on the spot price change. The estimates $\beta_{1,1}^{(1)}$ , $\beta_{1,1}^{(2)}$ , $\beta_{1,2}^{(2)}$ are all statistically significant at the 0.1% level, implying that the lagged futures price significantly influence the spot price movement for both regimes. Because $\beta_{1,1}^{(2)}$ (0.62) is greater than $\beta_{1,2}^{(2)}$ (0.12), the futures price tends to impact more in the recent past on the current spot price than the more distant past. In contrast, amongst the $\alpha_{2,1}$, $\alpha_{2,2}$ estimates, only $\alpha_{2,2}^{(1)}$ is statistically significant, at the 0.1% level, suggesting that within the first regime the distant past spot price change has a significant impact on the current futures price movement.

Table 5.5 also presents the estimates for the loading parameters. The statistical insignificance of $\theta_1^{(1)} = -0.03$ and $\theta_2^{(1)} = -0.06$ together with their negative sign indicate that the canola spot and futures prices follow a random walk without any apparent adjustment over Regime 1. In contrast, $\theta_2^{(2)}$ = 0.02 is statistically significant at the 5 percent level, suggesting the presence of an error correction mechanism in Regime 2 and the primacy of the canola futures market in adjusting and re-establishing the price equilibrium. Although this finding contrasts with the results of Brockman and Tse (1995), it does corroborate the general findings in Tse (1999), Yang et al. (2001), Lien and Tse (2002), and Zhong et al. (2004), who found that the futures market plays a more informational role maintaining the spot-futures price equilibrium.

Based on the estimates for $\theta_1^{(2)}$ and $\theta_2^{(2)}$ are the cointegrating vector [1,-1.05], the error correction term in Regime 2 for $\Delta S_t$ is $-0.01(S_{t-1} - 1.05 f_{t-1})$ and for $\Delta f_t$ is

$0.02(S_{t-1} - 1.05 f_{t-1})$ , (4.3b). The opposite signs of $\theta_1^{(2)}$ and $\theta_2^{(2)}$ , as well as opposite signs of $\gamma_1$ and $\gamma_2$, corroborate our expectation. The opposite signs of $\theta_1^{(2)}$ and $\theta_2^{(2)}$, as well as opposite signs of $\gamma_1$ and $\gamma_2$, indicate that any spot futures price deviations greater than the threshold will activate the error correction mechanism and bring about a reduction in the differential and for the equilibrium to become reinstated. If $S_t > f_t$, then the error correction term $-0.01(S_{t-1} - 1.05 f_{t-1})$ will apply and force $S_t$ to decrease and $f_t$ to increase, while if $S_t < f_t$, then the term $0.02(S_{t-1} - 1.05 f_{t-1})$ comes into effect as $S_t$ will increase and $f_t$ will decrease.

The Regime 2 error correction mechanism can also be interpreted in the market microstructure context. For a canola spot-futures price disequilibrium, a positive (negative) deviation implies the futures price is above (below) the long run price equilibrium level, so futures traders sell (buy) the futures contract, which results in a futures price fall (rise), and causing a negative (positive) futures returns in the next period. On the other hand, when the canola spot price is over (under) its price equilibrium, arbitragers would attempt to sell and buy the canola at its spot and futures markets, in turn causing the spot price to fall (rise), and leading to a negative (positive) return in the next period. Finally, by calculating the common factor weights via (4.4), the Canadian canola spot market contributes 33.3 percent and the futures market contributes 66.7 percent in both regimes to their price information discovery, which implies that market participants trading canola follow the futures price more than the spot market price in Canada.

### 5.6 Volatility Analysis

Studies of Koutmos and Tucker (1996), Tse (1999), Zhong et al. (2004), recognise that traders have a speculative drive to exploit temporary price differential between the spot and futures market. Since when price disequilibria occurs, trading should be intensified and this intensification is likely to result in greater volatility. In addition to the analysis at the price level and price return, therefore we examine the volatility in order to reveal information further regarding price change volatility. As illustrated in Tse (1999), Zhong et al (2004), and Srinivasan and Ibrahim (2012), the residuals from the threshold error correction model estimation is used for estimating the bivariate GARCH model. It is expected that the heteroscedasticity to exist on the residual series, thus the uncertain portion of the price information could be revealed. According to Srinivasan and Ibrahim (2012), the TVECM estimation as specified in (4.3a) and (4.3b) are still unbiased despite the error terms having a time varying variance. This two-step estimation is asymptotically equivalent to an aggregate procedure that estimates the TVECM and bivariate-GARCH jointly.

The presence of conditional heteroscedasticity was assessed by evaluating the multivariate Ljung-Box statistic, Hosking (1980), and the non-parametric rank-based test, Dufour and Roy (1985). The hypothesis of the $\varepsilon_{i,t}^2$ for i=1,2, (4.3) is tested which exhibits no conditional heteroscedasticity. This entails the conditional covariance matrix $\sum_t$ being time invariant and $\varepsilon_{i,t}^2$ independent of $\varepsilon_{i,t-1}^2$. Between the two tests, the rank-based test is preferred because it is immune to a noise process having heavy tails. The test results are presented in Table 5.6. Both the multivariate Ljung-Box and rank-based test results provide strong evidence against the no heteroscedasticity hypothesis and indicates the time-variation of $\varepsilon_{i,t}$ series.

**Table 5.6 Conditional Heteroscedasticity tests**

| For residuals $\varepsilon_{i,t}$ from (3a) and (3b) estimation | | |
|---|---|---|
| | Test Statistics | p-value |
| Multivariate Ljung-Box statistics | 690.55 | (0.0001) |
| Rank-based test | 1242.99 | (0.0001) |

Notes: All tests were carried out with $H_0$ that the noise process epsilon has no conditional heteroscedasticity against $H_1$ of time-invariant conditional covariance.

### 5.6.1 A Bivariate GARCH model

Our model used to examine volatility is analogous to the bivariate GARCH model in BEKK class of Chan et al. (1991), Zhong et al. (2004), Srinivasan and Ibrahim (2012), and many among others. An alternative to this approach is the dynamic conditional correlation (DCC) model of Engle (2002). For the DCC model, univariate GARCH models coupled with parsimonious parametric models are formulated to calculate the correlation. Thus the volatility effect across markets can be identified by examining the correlation magnitude.

We used the BEKK[20] GARCH model because it is more suitable for our analysis and more compatible to our empirical methodology. In fact, the DCC model is equivalent to a scalar BEKK which uses scalar instead of parameter matrix for the model coefficient[21]. It's formulation brings ease in modelling large

[20] As mentioned in Engle, R. F. and Kroner, K. F. (1995) Multivariate Simultaneous Generalized ARCH. *Econometric Theory* 11, 122-150.Engle and Kroner (1995), the original contribution of BEKK GARCH model attribute to Baba, Engle, Kraft, and Kroner thus their contribution refers to a BEKK class model.

[21] For ease of estimation, it is possible to impose a less general diagonal BEKK model i.e. A and B are diagonal matrices, for volatility modelling within a multivariate system. However, this type of alternative is not suitable for our case since the primary object of our interest is examining the volatility transmission across markets. The off-diagonal elements in matrices A and B are particularly used in capturing the volatility spillover effect.

system data. However, in our case with bivariate series, the rich parameterisation of BEKK is more conducive since more explicit description for the spillover and clustering effect provided. Also, a multivariate GARCH model in BEKK class is more compatible to serve as the conditional volatility equation for the threshold error correction model. The two-step approach is asymptotically analogous to the estimation of TVECM and multivariate GARCH jointly. Finally, the BEKK has the merit, unlike the VECH-class GARCH model of Bollerslev et al. (1988), of requiring fewer parameters to be estimated and imposing non-negativity restrictions on the covariance matrix, which brings the added convenience when interpreting the results.

If the conditional variance $\sigma_{i,t}^2 = Var(\varepsilon_{i,t} \mid F_{t-1})$ for i=1,2, where $F_{t-1}$ is the information set at time *t-1*, are the respective elements of the 2×2 conditional variance-covariance matrix $\sum_t$ at time *t*, then the bivariate-GARCH model is specified as:

$$\sum_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'\sigma_{t-1}\sigma'_{t-1}B + G'\varsigma_{t-1}\varsigma'_{t-1}G \qquad (5.5)$$

In (5.5), C is a 2×2 upper triangular matrix of intercepts, A is a 2×2 ARCH parameter matrix capturing the information shock, B is a 2×2 GARCH parameter matrix describing the volatility clustering and persistence, and G is a 2×2 parameter matrix depicting the volatility asymmetry with $\varsigma_{i,t-1} = \varepsilon_{i,t-1} I_{\varepsilon_{i,t-1}<0}$ where $I_{\varepsilon_{i,t-1}<0}$ is the indicator function having value of one when $\varepsilon_{i,t-1} < 0$. By expanding (5.5), we have the asymmetric BEKK model representing the bivariate volatility structure for the spot and futures prices:

$$\sigma^2_{s,t} = c_{s,t-1} + A^2_{11}\varepsilon^2_{s,t-1} + 2A_{11}A_{21}\varepsilon_{s,t-1}\varepsilon_{f,t-1} + A^2_{21}\varepsilon^2_{f,t-1} + B^2_{11}\sigma^2_{s,t-1} + 2B_{11}B_{21}\sigma_{s,t-1}\sigma_{f,t-1}$$
$$+ B^2_{21}\sigma^2_{f,t-1} + G^2_{11}\varsigma^2_{1,t-1} + 2G_{11}G_{21}\varsigma_{1,t-1}\varsigma_{2,t-1} + G^2_{21}\varsigma^2_{2,t-1}, \tag{5.6a}$$
$$\sigma^2_{f,t} = c_{f,t-1} + A^2_{12}\varepsilon^2_{s,t-1} + 2A_{12}A_{22}\varepsilon_{s,t-1}\varepsilon_{f,t-1} + A^2_{22}\varepsilon^2_{f,t-1} + B^2_{12}\sigma^2_{s,t-1} + 2B_{12}B_{22}\sigma_{s,t-1}\sigma_{f,t-1}$$
$$+ B^2_{22}\sigma^2_{f,t-1} + G^2_{12}\varsigma^2_{1,t-1} + 2G_{12}G_{22}\varsigma_{1,t-1}\varsigma_{2,t-1} + G^2_{22}\varsigma^2_{2,t-1}; \tag{5.6b}$$

To assess the source of price information uncertainty, the statistical significance of $A_{21}$ for $\sigma^2_{s,t}$ and $A_{12}$ for $\sigma^2_{f,t}$ need be examined to identify if the past futures and spot market information shock $\varepsilon^2_{f,t-1}$ and $\varepsilon^2_{s,t-1}$ have discernible effect on the present spot (futures) market volatility. Secondly, the estimated value and statistical significance of $B_{21}$ and $B_{12}$ in (5.6a) and (5.6b) are examined to reveal the magnitude of volatility spillover impacts. Finally, the value and significance of $G_{11}$ and $G_{22}$ a are assessed so as to identify the intermarket volatility asymmetry for both markets.

The estimated BEKK parameters for (5.6) are summarised in Table 5.7. The statistically significant estimates $A_{1,1}$ and $A_{2,2}$ indicates that for both spot and futures price volatilities they have uncertainties that arise from their own unexpected returns in the past. However, the estimated values of $A_{2,1}$ and $A_{1,2}$ indicates both coefficients are statistically indifferent from zero, which suggest the past unexpected return from each spot and futures price movement does not significantly affect each other.

## Table 5.7 The BEKK model estimating results

$$\sigma^2_{s,t} = c_{s,t-1} + A^2_{11}\varepsilon^2_{s,t-1} + 2A_{11}A_{21}\varepsilon_{s,t-1}\varepsilon_{f,t-1} + A^2_{21}\varepsilon^2_{f,t-1} + B^2_{11}\sigma^2_{s,t-1} + 2B_{11}B_{21}\sigma_{s,t-1}\sigma_{f,t-1} + B^2_{21}\sigma^2_{f,t-1} + G^2_{11}\varsigma^2_{1,t-1} + 2G_{11}G_{21}\varsigma_{1,t-1}\varsigma_{2,t-1} + G^2_{21}\varsigma^2_{2,t-1}, \quad (4.6a)$$

$$\sigma^2_{f,t} = c_{f,t-1} + A^2_{12}\varepsilon^2_{s,t-1} + 2A_{12}A_{22}\varepsilon_{s,t-1}\varepsilon_{f,t-1} + A^2_{22}\varepsilon^2_{f,t-1} + B^2_{12}\sigma^2_{s,t-1} + 2B_{12}B_{22}\sigma_{s,t-1}\sigma_{f,t-1} + B^2_{22}\sigma^2_{f,t-1} + G^2_{12}\varsigma^2_{1,t-1} + 2G_{12}G_{22}\varsigma_{1,t-1}\varsigma_{2,t-1} + G^2_{22}\varsigma^2_{2,t-1}; \quad (4.6b)$$

| Coefficients | $c_{s,t-1}$ | $c_{f,t-1}$ | $A_{11}$ | $A_{12}$ | $A_{21}$ | $A_{22}$ | $G_{11}$ | $G_{12}$ | $G_{21}$ | $G_{22}$ | $B_{11}$ | $B_{12}$ | $B_{21}$ | $B_{22}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Estimation | 0.003*** | -0.002 | 0.123*** | -0.001 | 0.005 | -0.063*** | -0.005 | -0.012 | 0.002 | 0.005 | 0.962*** | -0.002 | 0.028*** | 0.97*** |
| t-statistics | (9.219) | (-1.414) | (7.541) | (-0.939) | (1.336) | (-3.589) | (-0.864) | (-1.358) | (0.739) | (0.832) | (127.072) | (-0.619) | (3.348) | (56.043) |

Note: BEKK estimating results for the second moment of ICE canola futures and spot price. $c_{1,1}$ and $c_{2,2}$ are model intercepts, $A_{11}$ and $A_{22}$ describe the own-information shock effect of canola spot and futures volatility series. $A_{12}$ and $A_{21}$ indicate the arrival of information shock across markets. $B_{11}$ and $B_{22}$ measure the volatility persistence, and $B_{21}$ and $B_{12}$ describe the volatility spillover magnitude. $G_{1,1}$ , $G_{1,2}$ , $G_{2,1}$ and $G_{2,2}$ are parameters capturing the asymmetry of volatility evolution. $\varsigma_{i,t-1} = \varepsilon_{i,t-1} I_{\varepsilon_{i,t-1}<0}$ with $I_{\varepsilon_{i,t-1}<0}$ the indicator function which has value of one when $\varepsilon_{i,t-1} < 0$, '***','**','*', denote 0.1%, 1%, and 5% significance level, respectively. The maximum likelihood estimation approach is used during the parameter estimation.

The parameters $B_{11}$ and $B_{22}$ which reflect the feature of volatility clustering are highly significant and demonstrate the tendency for the spot (futures) price volatility to persist. However, the estimated parameters reflecting the volatility spillover effect produced mixed results. The estimated results show that only $B_{21}$ is significant, indicating that the volatility spillover from the futures to the spot market, and indicating that it is only futures price volatility significantly influencing spot market volatility. Finally, unlike the previous findings of positive return-volatility relationship for the commodity market, in our assessment there is no evidence supporting an asymmetric effect, potentially implying the market depth of canola futures market and the symmetric return-volatility relationship.

### 5.7 Summary and concluding remarks

In this chapter we examined the price information between canola spot and futures market in Canada. Following important findings are obtained based on the analysis from a TVECM-BEKK model. First, the identified cointegration between the canola spot and futures price indicates the two markets share a common stochastic trend in their long run. However, a threshold effect exists over the cointegration, in which canola spot-futures price equilibrium relationship appears time varying. Second, in examining the error correction mechanism of the two markets, it is found that over Regime 1 the canola spot and futures markets do not exhibit any significant adjustment to their price disparity. In comparison, the error correction mainly occurs in Regime 2 when the futures price change is dependent variable. Similar to Lien and Tse (2002), and Zhong et al. (2004), the present study’s results based on canola spot-futures price suggest that the futures market is more swift in accommodating the new price information. Additionally, the canola spot and futures market have relative contribution at about 33.3% and 66.7% respectively in maintaining their price equilibrium. In terms of uncertainty of price information where the second moment of canola spot-futures price indicate, the estimating results suggest that both markets have significant volatility persistence. But volatility evolution in the spot markets was partially a spillover from the futures markets, implying that the instability source from the futures price is impacting the canola spot price.

By examining the price interlinkage and volatility interrelation between the canola spot and futures markets, this study contributes to the literature on canola price discovery with a non-linear empirical methodology, which takes into account a potential threshold over the price adjustment to their long run equilibrium. Compared with previous linear models, the threshold error correction model is advantageous in reproducing the no-arbitrage band as well as structural change caused by a regime switching issue. Secondly, in examining the canola spot-futures price information association, this study developed a joint investigation procedure with the threshold error correction model and bivariate GARCH model for the canola price discovery, which is a methodological extension to previous studies of Khoury and Yourougou (1991) and Brockman and Tse (1995). However, the empirical work of the present study is limited to the Canada based canola price only. For further research it could be interesting to evaluate price discovery of the canola price from other regions for a comparison.

# Chapter 6. Conclusion

This thesis conducts empirical analyses based on GARCH models in different forms to examine their performance in evaluating derivatives of stock, currency, and commodity markets. The significance of evaluating the role of stochastic volatilities in time series modelling has been realised since Black(1976), Engle(1982), and Bollerslev (1986) due to the need to obtain more accurate asset pricing models. Three classes of derivatives were considered in the GARCH models application of the present study. These are (i) the European style S&P500 index option, (ii) the European style foreign exchange option, and (iii) the Canadian canola commodity futures. In the valuation of options, the valuation exercise in Chapter 3 and 4 endeavoured to simulate model option price by estimating a stochastic volatility structure for the historical underlying price that accommodates stylised facts in the respective stock and foreign exchange markets. In evaluating canola futures, the focus in Chapter 5 was on its interrelation with underlying spot market price movement and the volatility multidimensionality reflecting price change uncertainty such as spillover effect.

## 6.1 S&P500 option pricing using GARCH models

For the first empirical inquiry the Chapter 3 has had the valuation of the S&P500 options under the GARCH volatility process. Conventional models such as Black-Scholes model considers the use of instantaneously constant volatility during the option valuation over any terms to expiration, which is deemed as a critical limitation of the model and has been generalised onto the alternative form with updating of implied volatilities, or volatility representation with time varying dynamic. The current study furthers the later line by considering the stochastic volatility described by a GARCH process, which could estimate the time varying volatilities using historical underlying price data and circumvent the computational burden of continuous models that infer the implied volatility variable from large amount option data over time and exercise price.

By reviewing the related literatures, numerous GARCH specifications have been developed to date. Unfortunately, existent studies yield conflicting findings in respect of the best performing models and alternative limitations such as structural breaks caused model estimation robustness issues, inconsistent benchmark models, and inconsistent loss function use across empirical inquiries. More importantly, these models' advances and ones' own numerical methods result unavoidably in implementing barriers to their application. Among existent methodologies it remains void considering the parsimonious form of these commonly applied models for option pricing. In the meantime, essential stylised facts of volatility asymmetry, clustering, and mean-reversion are retained to investigate if these parsimonious models remain effective in option valuation. In a practical consideration, an assessment is of considerable importance as estimating work becomes readily accessible through standard statistical packages [22], which bring ease of obtaining the model estimates.

Given these, the Chapter 3 conducted empirical analysis which builds on previous empirical studies on the S&P index that propose a variety of forms

[22] To our best knowledge the statistical software available for estimating our GARCH models at least including R, MATLAB, S-plus, and STATA to date.

representing volatility structure, including the ordinary GARCH model of Bollerslev (1986), the exponential GARCH model of Nelson (1991), the nonlinear asymmetric GARCH model of Engle and Ng (1993), the threshold GARCH model of Glosten et al. (1993), and the closed-form asymmetric GARCH model of Heston and Nandi (2000). These models have heavy use in the financial application and their most parsimonious forms are considered in this study. Also these models in asymmetric form were applied in present study because it is the leverage effect that critically matters in modelling stock price volatility (Black 1976). The research objective was to assess the GARCH class stochastic volatility models against the continuous time constant volatility models in their S&P500 index option pricing performance. The subsequent analysis attempted to answer the research questions: (i) of what form the GARCH model has the least valuation errors, and (ii) whether the source of inaccuracy of constant volatility models arises from its inaccurate measurement of time-varying volatility. The performance of alternative GARCH models was evaluated based on their valuation error defined as the difference between the simulated and market option price, and also their valuation errors relative to the Black-Scholes and Gram-Charlier model which are benchmarks used in this assessment.

The main contribution of this first empirical chapter was in the pursuit of GARCH option pricing models possessing simplicity but retaining essential volatility properties. The valuation results show that the GARCH model of Bollerslev (1986) with simplest form yields the least overall valuation errors, which is primarily attributed to its optimal performance in valuing the short-maturity options. Also, like the previous findings of Christoffersen and Jacobs

(2004b), Hsieh and Ritchken (2005), asymmetric non-linear GARCH models remain to be the preferred form amongst the GARCH variants. For the short term near-the-money contracts, it is the EGARCH model of Nelson (1991) which performs best, since its news impact function is the most sensitive to the information shock.

Besides the pursuit of effective parsimonious GARCH model in option pricing, another contribution this first empirical analysis endeavoured to provide is the consistency applied over the methodological approach. Unlike previous studies neglecting potential structural break over the time series which could result in variable model parameters, the robustness of the simulation result was enhanced with model estimates obtained from a structural break free sample span. Moreover, unlike previous studies with inconsistent use of benchmark models and loss functions, during the comparative analysis of the first analytical chapter the two benchmarks were employed to have more fledged criteria, i.e. a Black-Scholes model corresponds to the special case of GARCH model with constant volatility, and a Gram-Charlier model parallels the GARCH model with a fat-tail and skewed distribution. Overall, the Gram-Charlier model performs well, and certainly better than the Black-Scholes model because of accommodating both skewness and kurtosis. This suggests that even for the constant volatility option pricing models, accounting for fat-tail is beneficial because of its versatility in more fully representing the underlying price distribution.

## 6.2 Foreign exchange option pricing using component GARCH models

Chapter 4 evaluated the European style foreign exchange options written on exchange rate of EURUSD, GBPUSD, and GBPEUR with the time-varying volatility process. Like the Black-Scholes model, the earliest model developed for foreign exchange options pricing is proposed in Garman and Kohlhagen (1983), in which the most original ideal conditions are retained including the constant volatility assumption. Duan and Wei (1999) proposed the first GARCH type currency option pricing model, in which time-varying volatility under a non-linear GARCH process is applied. Many others such as Bollen and Rasiel (2003) and Harikumar et al (2004), Aduda (2011), Ulusoy and Onbirler (2014) and Bhat and Arekar (2016) further this line and investigated the currency option valuation performance under the GARCH framework amongst alternative foreign exchange markets.

In reviewing the related literature, it is found that existent researches failed to identify the best performing GARCH representation in the measurement of symmetry, asymmetry, and the mean-reversion effect in the foreign exchange volatilities. The symmetry refers to the symmetric return-volatility relationship of the foreign exchange rate, given the fact that for any bilateral exchange rate the return from one currency always results in the equal loss for another. In comparison, the ‘Leverage effect’ in the foreign exchange market can also arise given the noise traders speculation and hedging activities through the foreign exchange derivatives. Lastly, the mean-reversion nature over the variation of exchange volatilities also raised further complexities. Foreign exchange market deterministic theory such as the law of one price or purchasing power parity highlight an important distinction between the short and long run behaviour of volatility in the following way: in case that new information arrived, over the short time horizon the respective currency is always likely to respond timely given its most active trading nature over other financial markets. However, such a response also accompanies with short time decay. Over the long time horizon, it is the price level differential of the two countries of the given exchange rate that determine the long run exchange rate movement. Therefore a volatility representation accommodating both transitory and persistent volatility characters implicitly offers appropriate description to this exchange rate evolution.

A non-affined component GARCH representation proposed in Christoffersen et al. (2012a) was considered in this chapter, which nests both symmetric and asymmetric GARCH models and has a volatility evolution described by a component structure. A two components structure, with a short-run volatility component capturing the transitory effects of unexpected return on the conditional volatility and a long-run component representing persistency and reflecting the impact of mean reversion on long run volatility, provides a more effective and richer representation of the volatility behaviour. In this way, the overall formulation provides a more effective and richer representation of the volatility behaviour.

By this, the empirical analysis in Chapter 4 attempted to answer the research questions that (i) if a GARCH model with and without volatility asymmetry yields significant difference in the foreign exchange option valuation, and (ii) if the conventional GARCH model accommodating the long run volatility component could give improved performance.

Using the data of three pairs of exchange rates with currency options issued in Euronext Amsterdam exchange, it was shown the single component GARCH model, which is exclusively applied in Duan and Wei (1999), Bollen and Rasiel (2003), Harikumar et al. (2004), Ulusoy and Onbirler (2014), and Bhat and Arekar (2016), fail to have explicit measurement of the volatility behaviour. But the significant improvement could be achieved through a two-component GARCH representation in foreign exchange volatility modelling and subsequent option pricing.

The empirical study in Chapter 4 made the following contributions. First, in contrast to Hsieh (1988), Bollerslev et al. (1992), Andersen et al. (2001), and Maya and Gomez (2008), who claim that the foreign exchange market exhibits a symmetric return-volatility linkage in responding to new information from unexpected return, it was shown that this effect is mainly confined to the long run. Over the short time horizon there exists volatility asymmetry. And this is most pronounced for the EURUSD and GBPEUR exchange rate. The implicit cause could be attributed to the base currency effect. The Euros and British pounds have comparatively less economic significance and scale than U.S. dollars. Therefore, for any significant rise and decline on the given exchange rate could cause an asymmetric response from the respective currency investors. And this base currency effect would become less significant to currency pairs with economic significance relatively less distinctive from each other as the GBPEUR exchange rate.

Secondly, this practice complements existing literature using component GARCH models for foreign exchange option pricing. The estimating results show strong evidence for a pronounced persistence effect in the long run. The component GARCH model always provides a better fit of the underlying exchange rate than single volatility component models. Furthermore, at times when short- and long-run volatility components are built into the conventional

GARCH models, the component framework offers greater versatility and extra control in the option price simulation. During the comparative analysis amongst these models the component GARCH models exhibit dominating performance over the remaining models, particularly in the valuation of currency options with long term to expiration.

By and large, Chapter 4 reveals whether alternative component models of currency options could benefit from moving beyond the conditional GARCH(1,1) framework and the Black-Scholes class models. According to the Bank of International Settlements report, the global foreign exchange market represents the largest financial market with turnover amounts to $5 trillion in 2016, with the financial derivatives heavily used on a daily basis. The size of foreign exchange markets reflects the rising requirement for the management of currency risk exposure in view of an aggregated global financial market. Therefore the investigation for the more representative exchange rate modelling formulation and respective options valuation approach deserves considerable effort. It is expected that the comparative assessment amongst the component models and conventional GARCH models as well as the Garman-Kohlhagen model provide useful guidance to practitioners and market participants in respect of the more accurate approaches for pricing and trading foreign exchange derivatives.

**6.3 Canola futures evaluation of its information content with spot price**

Chapter 5 studied commodity futures in relation to its underlying spot price movement, and focused particularly on canola futures in Canada. Canada constitutes one of the most important parts in global agriculture due to its agricultural production. Canola, which is a contraction of ‘Canada oil’, represents one of its most primary commodities with the largest trading volume and the least government regulation. In recognition of the active role of canola futures in the price discovery process relative to other commodities such as wheat and barley, this chapter attempted to explore the informational role and content of its futures markets in relation to its spot market subject to price change and price volatility, although few studies have been done in its price discovery process.

When review the previous researches on price information interdependency of the canola markets, the multidimensionality of the canola of the spot-futures price system raises a different set of issues. In this exploration, this chapter investigated the scale and extent of the two market interdependency in a world of time-varying volatility. The Johansen procedure, used for assessing long-run price equilibrium, has the implicit assumption of a dynamic constant equilibrium relationship. This linear formulation in fact fails to account for potential structural change in the adjustment process due to transaction costs and any significant event, which create a no-arbitrage threshold and discontinuous trading. The existence of an inactively traded price zone can be more fully represented by a threshold cointegration and error correction model for investigating the price interdependence between canola spot and futures price. The empirical inquiry in Chapter 5 attempted to answer the research question whether the threshold cointegration and error correction model procedure provide a more robust assessment on price interdependence between canola spot and futures price. Further, subsequent assessment in this chapter applied a BEKK type GARCH model to investigate the extent and nature of the uncertainty of price information interacted between the canola spot and futures markets.

A three-stage procedure was adopted in evaluating the canola futures. First, a unit root and cointegration analysis were performed to investigate the existence of a price equilibrium. Here, the Hansen and Seo (2002) test examines any presence of any threshold over the cointegration and error correction process, or whether arbitrage trading is activated only for an opportunity profit sufficiently exceeding a certain threshold to compensate the incurred transaction costs. Second, the equilibrium process was approximated through a threshold error correction model, which not only describes the adjustment process of the spot-futures price differential but permits the regime-switching pattern of price equilibrium to be identified and the no-arbitrage band to be measured. Finally, a multivariate GARCH model in the BEKK class was used to evaluate the price information uncertainty, where the estimation is based on the residual series from ECM, which reflects the uncertainty of the spot-futures price information.

The empirical study in Chapter 5 on canola spot-futures price made following contributions. Firstly, in view of existing studies investigating the canola spot and futures price relationship, this analytical chapter contributed to the study of canola spot and futures price behaviour with analysis applying a non-linear threshold error correction model. The empirical findings show that the canola spot and futures prices exhibit cointegration with a threshold, suggesting a time-varying price equilibrium with two regimes. While in the first regime neither the canola spot nor the futures price exhibit any significant adjustment to their price disparity, the error correcting mechanism appears mainly in the second regime where the futures market is the more swift in impounding the arrival of the new information. It was demonstrated that, at least for canola, the merit of applying a non-linear threshold error correction model with its properties of a no-arbitrage band as well as a regime-switching price equilibrium. This finding could be useful to the arbitragers and market traders who are concerned with the dynamic of spot-futures price disparity over time and have concurrent involvement in trading and regulating markets.

Secondly, this exercise undertook the first step jointly analysing canola spot-futures price volatility behaviour and interactions. Futures contracts have been primarily used in commodity markets to allow market participants to guard against adverse price movement. With an assessment of volatility interrelation between the two markets, agents involved in any markets could better understand any uncertainties and adverse price fluctuations beforehand and thus reduce their trading risk. The empirical results endorse Bauwens et al (2006), Caporin and McAleer (2009), Assefa et al(2015) that the BEKK-GARCH remains be the most effective variant for assessing volatility transmission and particularly the spillover effect. It showed both the spot and futures price exhibiting pronounced volatility persistence, but there is volatility spillover from the futures market to the volatility movement in the spot market.

### 6.4 Concluding remarks and further research

Overall, the three studies implemented in this thesis have focused on the application of GARCH models in accompany with Monte Carlo simulation to have more realistic evaluation of respective derivatives. In the valuation of the S&P 500 options, although the Gram-Charlier model providing an improvement in the distribution measurement over the classic Black-Scholes formula and having advantages of analytical solution and computational ease, the empirical results indicate that incorporating time-dependent heteroskedasticity in the volatility modelling is critical as valuation errors are significantly reduced. For the simulated option pricing model based on a GARCH volatility process, it is desirable to extend the application to effective option hedging techniques as well as to more complex designs such as exotic options and also American-style options.

In the valuation of the foreign exchange options, it has been shown the dominance of the two component GARCH style model for actively traded currency pairs. Also the symmetric relationship between exchange rate movement and exchange rate volatility arise primarily in the long run, during the short time horizon the transitory asymmetry could rise in the Forex market due to a base currency effect and asymmetric response from Forex market agents to the arrival of new information. For further research it would be interesting to investigate these models' performance in less actively traded currencies and new established foreign exchange derivatives markets. In addition, although the GARCH models provide improved realism in characterising the stochastic volatilities, the increased complexity also raises potential difficulties and limitations in establishing the models particularly with analytical tractable solutions. In this regard, it would always be feasible to consider the numerical methods as resort in the valuation of financial derivatives given more explicitly characterised volatility. As computers continuously quicken their computational speed at a remarkable pace, and the growing studies on the computational algorithms for solving complex structures, the requirement for any analytically tractable solutions tend to diminish, although expositional barriers could still remain.

Finally, the third empirical inquiry evaluated the price discovery of canola futures versus its cash markets. Exploring the threshold framework, this chapter contributed to the existent literature by quantifying the non-linearity of the spot-futures price equilibrium relationship of the canola market. However, although this study characterised this equilibrium over time, one could further extend the limitation of this methodology by establishing an error correction model with price disparity being measured as an time-varying process. Extending the current research scope onto other futures markets by investigating whether there is an alternative interrelationship between the cash and futures market would be also interesting. Lastly, Chapter 5 chosen the canola market given the basis of its largest open interest and trading liquidity. Conducting price discovery on alternative commodities which lack liquidity and investigating its information content could be important as well. Studies on this investigation subject to further price discovery practice.

## Appendix

### A.1 The analytical solution of Heston and Nandi (2000) model

Under the physical measure P, Heston and Nandi (2000) assume the logarithmic spot price of underlying asset has the following GARCH process:

$$\begin{aligned} &\ln(S_t) = \ln(S_{t-1}) + r_f + \lambda h_t + \sqrt{h_t}\varepsilon_t \\ &h_t = \beta_0 + \sum_{i=1}^{p}\beta_t h_{t-i} + \sum_{i=1}^{q}\alpha(\varepsilon_{t-i} - \gamma_i\sqrt{h_{t-i}})^2 \end{aligned} \qquad \text{A 2.1}$$

where z (t) is a standard normal disturbance. The main difference is that $\gamma_i$ here governs the skewness (asymmetry) of the log-spot price distribution, which reflects the leverage effect in the equity asset. The conditional variance and logarithmic price has the following relationship,

$$Cov_{t-1}(h_{t+1}, \ln S_t) = -2\alpha\gamma_1 h_t \qquad \text{A 2.2}$$

The positive parameters α and $\gamma_1$ result in a negative correlation between volatility and log spot price. However, the equations A 2.1 are not risk-neutral. To price options using it one needs modify it in the following form,

$$\begin{aligned} &\ln S_t = \ln S_{t-1} + r_f - \frac{1}{2}h_t + \sqrt{h_t}z_t \\ &h_t = \beta_0 + \sum_{i=1}^{p}\beta_i h_{t-i} + \sum_{i=2}^{q}\alpha_i\left(\varepsilon_{t-i} - \gamma_i\sqrt{h_{t-i}}\right)^2 + \alpha_1\left(\varepsilon_{t-i} - \gamma_i^*\sqrt{h_{t-i}}\right)^2 \end{aligned} \qquad \text{A 2.3}$$

where $z_t^* = z_t + \left(\lambda + 1/2\right)\sqrt{h_t}$ , and $\gamma_1^* = \gamma_1 + \lambda + 1/2$ . To allow the process mean-reverting, it is needed that $\beta_i + \alpha\gamma^2 < 1$ . And the term $z_{t-i} - \gamma_i\sqrt{h_{t-i}}$ captures the variance persistence. The closed-form solution with one lag is given as,

$$C_{HN} = e^{-r_f T} E_t^*\left(S_{t+T} - X, 0\right) \\ = S_t W_1 - X e^{-r_f T} W_2$$ A 2.4

where $W_1$ and $W_2$ take the form as,

$$w_1 = \frac{1}{2} + \frac{e^{-rT}}{\pi S_t}\int_0^\infty \mathrm{Re}\left[\frac{X^{-i\phi} f^*(i\phi+1)}{i\phi}\right] d\phi \\ w_2 = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{X^{-i\phi} f^*(i\phi)}{i\phi}\right] d\phi$$ A 2.5

'Re' refers to the real part of the generating function $f^*(\phi)$ in a risk-neutral world, where

$$f^*(\phi) = S_t^{\phi} \exp\left(A_t + B_t \sigma_{t+1}^2\right) \\ A_t = A_{t+1} + \phi r_f + B_{t+1}\gamma_3 - \frac{1}{2}\log(1 - 2\alpha B_{t+1}) \\ B_t = \phi(\lambda + \gamma_1) - \frac{1}{2}\gamma_1^2 + \beta B_{t+1} + \frac{1/2(\phi - \gamma_1)^2}{1 - 2\alpha B_{t+1}}$$ A 2.6

## A.2 The GARCH model Simulation in MATLAB

The code below for Black-Scholes formulation modelling and GARCH model option pricing using Monte Carlo simulation has original contribution from www.volopta.com providing the permission from the original contributor of Anon (2010), the author give further development for the remaining asymmetric GARCH models. The coding of variance reduction technique has the original contribution from Cerrato (2012).

## Black-Scholes formula for call and Put

```
BSC = @(s,K,r,d,v,T) (s*exp(-d*T)*normcdf((log(s/K) +(r-d+v^2/2)*T)/v/sqrt(T))...
- K*exp(-r*T)*normcdf((log(s/K) + (r-d+v^2/2)*T)/v/sqrt(T) - v*sqrt(T)));

BSP = @(s,K,r,d,v,T) (K*exp(-r*T)*normcdf(-(log(s/K) + (r-d+v^2/2)*T)/v/sqrt(T)...
    + v*sqrt(T)) - s*exp(-d*T)*normcdf(-(log(s/K) + (r-d+v^2/2)*T)/v/sqrt(T)));
```

## The ordinary GARCH model of Bollerslev (1986)

```
  n=107; T = n/360; S=1920.24; rf=0.0338e-4 ;  omega  = 2e-6;     alpha1= 0.103146; beta1  =0.873104;
Nsims   = 50000;
Spot    = S;
K       = 1850;
S1      = zeros(n,Nsims);
S1(1,:) = Spot;
S2      = zeros(n,Nsims);
S2(1,:) = Spot;
T       = n/360;
e(1)    = 0;
h(1)    = variance;
for i=1:Nsims;
          for t=2:n
                      h(t) = omega + alpha1*(e(t-1)-0.5*h(t-1))^2 + beta1*h(t-1);
     e(t) = sqrt(h(t))*(randn(1));
                      S1(t,i) = S1(t-1,i)*exp(0-0.5*h(t)+e(t));
   end
             ST1(i) = S1(end,i);
end
GARCHCall = exp(-rf*T)*mean(max(ST1-K,0));
GARCHPut  = exp(-rf*T)*mean(max(K-ST1,0));
```

## The exponential GARCH model

```
variance =1.85266E-05;
sigma = sqrt(variance)*sqrt(360);
n=121; T = n/360; S=1987.01; rf=0.0369e-4;   omega  =-0.36068 ;
alpha1= -0.20936; beta1  = 0.96067; gamma1= 0.10716;

Nsims   = 50000;
LnVariance = log(variance) ;
Spot    = S;
```

```
K       = 1850;
S1      = zeros(n,Nsims);
S1(1,:) = Spot;
S2      = zeros(n,Nsims);
S2(1,:) = Spot;
T       = n/360;
e(1)    = 0;
h(1)    = variance;
matrix0 = [];
matrix1 = [];

for i=1:Nsims;
          for t=2:n
      h(t) =exp( omega + beta1*log(h(t-1)) + alpha1*(e(t-1)/sqrt(h(t-1))-0.5*sqrt(h(t-1)))+...
               gamma1*(abs(e(t-1)/sqrt(h(t-1))-0.5*sqrt(h(t-1)))-sqrt(2/pi)));
                     e(t) = sqrt(h(t))*randn(1);
                     S1(t,i) = S1(t-1,i)*exp(0-0.5*h(t)+e(t));
          end
          ST1(i) = S1(end,i);
end
      GARCHCall =  exp(-rf*T)*mean(max(ST1-K,0));
      GARCHPut  =  exp(-rf*T)*mean(max(K-ST1,0));
```

## The GJR-GARCH model

```
Nsims   = 50000;
Spot    = S;
K       =1850;
S1      = zeros(n,Nsims);
S1(1,:) = Spot;
S2      = zeros(n,Nsims);
S2(1,:) = Spot;
T       = n/360;
e(1)    = 0;
h(1)    = variance;
matrix0 = [];
matrix1 = [];
n=113;    T      =n/360;    S=1929.8; rf=0.3607e-4;    omega =4e-6;
alpha1=0; beta1  =0.840075; gamma1=0.248599;

% Simulate paths for the stock price, and retain the terminal prices S(T).
for i=1:Nsims;
          for t=2:n
      if e(t-1) < 0,
         h(t) =  omega + alpha1*((1-gamma1)^2)*((e(t-1)-0.5*h(t-1))^2) + beta1 * h(t-1);
      else
```

```
            h(t) = omega + alpha1*((1+gamma1)^2)*((e(t-1)-0.5*h(t-1))^2) + beta1 * h(t-1);
        end
           e(t) = sqrt(h(t))*randn(1);
           S1(t,i) = S1(t-1,i)*exp(0 - 0.5*h(t-1) + e(t));
           ST1(i) = S1(end,i);
    end
end
      GARCHCall =  exp(-rf*T)*mean(max(ST1-K,0));
      GARCHPut  =  exp(-rf*T)*mean(max(K-ST1,0));
```

## The nonlinear asymmetric GARCH model

```
Nsims   = 50000;
Spot    = S;
K       =1850;
S1 = zeros(n,Nsims);
S1(1,:) = Spot;
S2 = zeros(n,Nsims);
S2(1,:) = Spot;
e(1) = 0;
h(1) = variance;
matrix0=[];
matrix1=[];
n=121; T = n/360; S=2114.15;rf=0.1300e-4;  omega = 3e-6;
alpha1=0.061129;  beta1 =0.717497; gamma1=1.757981;
for i=1:Nsims;
          for t=2:n
                      h(t) = omega + alpha1 * h(t-1) * (e(t-1) -gamma1)^2 + beta1*h(t-1);
                      e(t) = randn(1);
                      S1(t,i) = S1(t-1,i)*exp(0 - 0.5 * (h(t)) + e(t) * sqrt(h(t)));
          end
          ST1(i) = S1(end,i);
end
   GARCHCall =  exp(-rf*T)*mean(max(ST1-K,0));
   GARCHPut  =  exp(-rf*T)*mean(max(K-ST1,0));
```

## Variance-reduction technique

```
gbmC    = @(s,K,r,sigma,T,Nsims)(mean(exp(-r*T).* max((s.* exp((r-sigma.^2/2)*T + sigma.*sqrt(T).*
(randn(Nsims,1))))-K,0)));
 gbmP    = @(s,K,r,sigma,T,Nsims)(mean(exp(-r*T).* max( K -(s.* exp((r-sigma.^2/2)*T + sigma.*sqrt(T).*
(randn(Nsims,1)))),0)));
%priliminary variables of the control variate technique for the MC
%simulation of European CALL;
avCm  = exp(-rf*T)*((GHcall_matr1+GHcall_matr2)/2);
gbmCm = exp(-r*T).* max((s.* exp((r-sigma.^2/2)*T + sigma.*sqrt(T).* (randn(Nsims,1))))-K,0);
covC1 = (avCm  - mean(avCm))*(gbmCm - mean(gbmCm));
varC1 = (gbmCm - mean(gbmCm))'*((gbmCm - mean(gbmCm)));
betaC = covC1/varC1;

priliminary variables of the control variate technique for the MC
simulation of European PUT;
avPm  = exp(-rf*T)*((GHput_matr1+GHput_matr2)/2);
gbmPm = exp(-r*T).* max(K - (s.* exp((r-sigma.^2/2)*T + sigma.*sqrt(T).* (randn(Nsims,1)))),0);
covP1 = (avPm  - mean(avPm))*(gbmPm - mean(gbmPm));
varP1 = (gbmPm - mean(gbmPm))'*((gbmPm - mean(gbmPm)));
betaP = covP1/varP1;

avGHcall      = exp(-rf*T)*mean((GHcall_matr1+GHcall_matr2)/2);
gBmCall       = gbmC(Spot,K,rf,sigma,T,Nsims);
BSCall        = BSC(Spot,K,rf,sigma,T)
cvGHcall      = avGHcall - betaC*(gBmCall-BSCall)
avGHput       = exp(-rf*T)*mean((GHput_matr1+GHput_matr2)/2);
gBmPut        = gbmP(Spot,K,rf,sigma,T,Nsims);
BSPut         = BSP(Spot,K,rf,sigma,T);
cvGHput       = avGHput - betaP*(gBmPut-BSPut);
```

## A.3 The influence from the financial crisis 2007-2008 to the GARCH model performance

*There have been* ***some major economic events*** *and* ***new policy shifts*** *in the financial markets, the structural breaks might have influenced the accuracy of the data projection and predictions, hence the formulation of your hypotheses and the results. Especially,*

*A detailed discussion of some significant historical perspectives of the ample period covers some significant events, such as the UK'S join and withdrawal from the ERM, which have implications for the exchange rate regimes. Importantly, it covers the financial crisis of 2007-2011. Please discuss how these significant events may have affected the exchange rate regime and stock market condition, and commodities markets, and how these impacts have been taken into evaluating forecasting accuracy for your empirical analyses. It is important to address these issues, because it would help to identify more accurate forecasting methods for each of these markets.*

According to the comments mentioned, this additional assessment investigated the effect of the economic events of the financial crisis 2007-2008 in order to examine any difference occurred during the model estimating process and therefore have a more thorough understanding of accurate forecasting methods based on GARCH type volatility models. It should be mentioned beforehand the events of UK join and withdrawal from ERM occurred some time ago (around 1990 and 1992), which is earlier than the earliest starting point of the original data sample of the present study. This additional assessment mainly focused on the financial crisis 2007-2008 and its period around 2007-2011 and examine any notable difference of the volatility behaviours.

The financial crisis is considered as the worst crisis since the global depression. Figure A.1 plots the historical daily index level of the original data sample between 1996 to 2016. As the figure illustrates, the decline of the index level starts from 2007 and reaches its peak in the September 2008, which corresponds to the time point of the collapse of the Lehman Brothers investment bank. Afterwards, the S&P500 index level experiences a further significant drop to its lowest level at about 676 since 1996.

**Figure A.3.1 S&P500 Index Level (1996-2016)**


Therefore in order to examine the influence of the financial crisis to the model performance in terms of volatility dynamic representation, the first data sample from 2003 June to the 2007 August was taken, as the sample of prior crisis period. This period was chosen because it starts from the time point right after the 2002 U.S. stock market clash and the 2003 war on Terror and Iraq War. Therefore one could isolate the influence from any previously occurred market events. As the figure illustrates, the periods between 2003 June to 2007 August has a relatively stable economic upturn trend without any significant index change variation. It is expected the model estimates obtained from this segment could provide us a reasonable benchmark relative to the financial crisis period model estimation. Afterwards the period from the end of August 2007 to the end of 2011 was picked as the sample period corresponding to the

**Table A.3.1 GARCH models estimation before and during the financial crisis 2007-2011**

| | $\omega_1$ | $\alpha_1$ | $\beta_1$ | $\gamma_1$ | $\lambda$ |
|---|---|---|---|---|---|
| **Prior-to-Crisis** (August 01, 2003 to August 30, 2007) | | | | | |
| E-GARCH | 5e-5 | -7.77e-2 | 0.99 | 0.074 | |
| | (7.4e-3) | (-2.98) | (7147) | (2.31) | |
| GJR-GARCH | 3e-6 | 0.00 | 0.88 | 0.12 | |
| | (4.45) | (0.00) | (20.86) | (2.88) | |
| NA-GARCH | 3e-6 | 0.039 | 0.608 | 2.73 | |
| | (9.04) | (4.25) | (23.52) | (4.43) | |
| HN-GARCH | 3.51e-6 | 1.46e-6 | 0.53 | 4.88e+2 | 7.21 |
| **During-the-Crisis** ( September 03, 2007 to December 30, 2011) | | | | | |
| | $\omega_1$ | $\alpha_1$ | $\beta_1$ | $\gamma_1$ | $\lambda$ |
| E-GARCH | -0.216 | -0.15 | 0.97 | 0.122 | |
| | (-42.24) | (-8.47) | (4.92e+4) | (7.62) | |
| GJR-GARCH | 3e-6 | 0.00 | 0.905 | 0.160 | |
| | (2.21) | (0.00) | (67.37) | (3.411) | |
| NA-GARCH | 4e-6 | 0.069 | 0.79 | 1.37 | |
| | (8.67) | (7.08) | (25.61) | (8.51) | |
| HN-GARCH | 2.5e-80 | 7.29e-6 | 0.77 | 164.4 | 0.67 |

Maximum Likelihood Estimates of GARCH (1,1) specifications using S&P500 index log-return from 01/August/2003 to 30/August/2007 for the Prior-to-Crisis sample, and from 03/September/2007 to 30/December/2011 for the During-the-Crisis sample. t-values for each parameter is reported in parentheses. $\omega_1$, $\alpha_1$, $\beta_1$, respectively refer to the constant parameter, information shock parameter, and persistence parameter. $\lambda$ in the HN-GARCH model is the unit risk premium . $\gamma_1$ is asymmetry parameter examining the leverage effect.

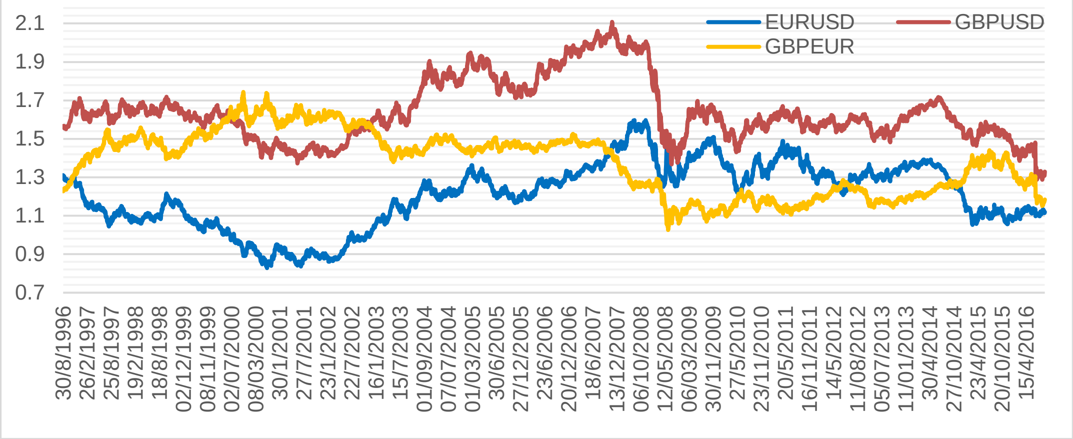
financial crisis. Models were estimated over two separate samples in order to identify any significant difference from the estimating results. Also both samples were restricted within a time span of about 4 years in order to have a more parallel comparative analysis.

Table A.3.1 summarises the model estimating results, with models presented in Table 3.1 of Chapter 3. As the models show, the difference appears in the $\alpha_1$ which is coefficient for the information shock term. Their estimated values are significantly higher during the crisis stage, which suggests the higher unexpected return over the crisis period. The persistence parameter which is represented by $\beta_1$ , also shows increased values with the exception of the exponential GARCH, which suggests that the persistence and clustering of the volatility are also stronger with any large variations persisting longer than the volatility fluctuation in the prior-to-crisis sample does. With respect to the leverage effect parameters, the estimated values show the mixed results but if one used the NA-GARCH as benchmark which has the best fitting of data, suggesting this leverage effect causes less asymmetric influence to the volatility evolution. A potential explanation would be that comparing to market participants' attitude to stock price decrease in the prior-to-crisis stage. During the crisis time any negative returns and stock decrease cause relatively less impression to the investors and other market participants. Lastly, for the Heston-Nandi GARCH model the unit premium parameter $\lambda$ shows the much smaller value of 0.67 in the crisis stage in comparison to its value of 7.21 in the prior-to-crisis time, indicating the potentially decreased risk premium during the crisis period.

Figure A.3.2 displays the variation of exchange rates of EURUSD, GBPUSD, and GBPUSD from 30/Aug/1996 to 01/Sep/2016, which are examined in Chapter.4 subject to their volatility modelling and currency option valuation performance. As the figure illustrates, it appears that both the EURUSD and GBPUSD exchange rates have a similar movement since both Euros and GBP currencies are exchanged for U.S. Dollars. In comparison, the GBPEUR exchange rate evolves in an opponent manner which seems dissimilar to the other two exchange rates. However, a common pattern appeared in the figure is that around the time point of September 2008 when the financial crisis turned into a full-blown international banking crisis, there are significant decreases in the level of all three exchange rates. This decrease is particularly strong in the GBPUSD exchange rates. However, it is less pronounced in the GBPEUR exchange rate as it is not directly linked to the U.S. Dollar. Moreover, this exchange rate fall approaches to its end at a timepoint around April of 2009 then all three exchange rates exhibit a certain rebound from their historical low level.

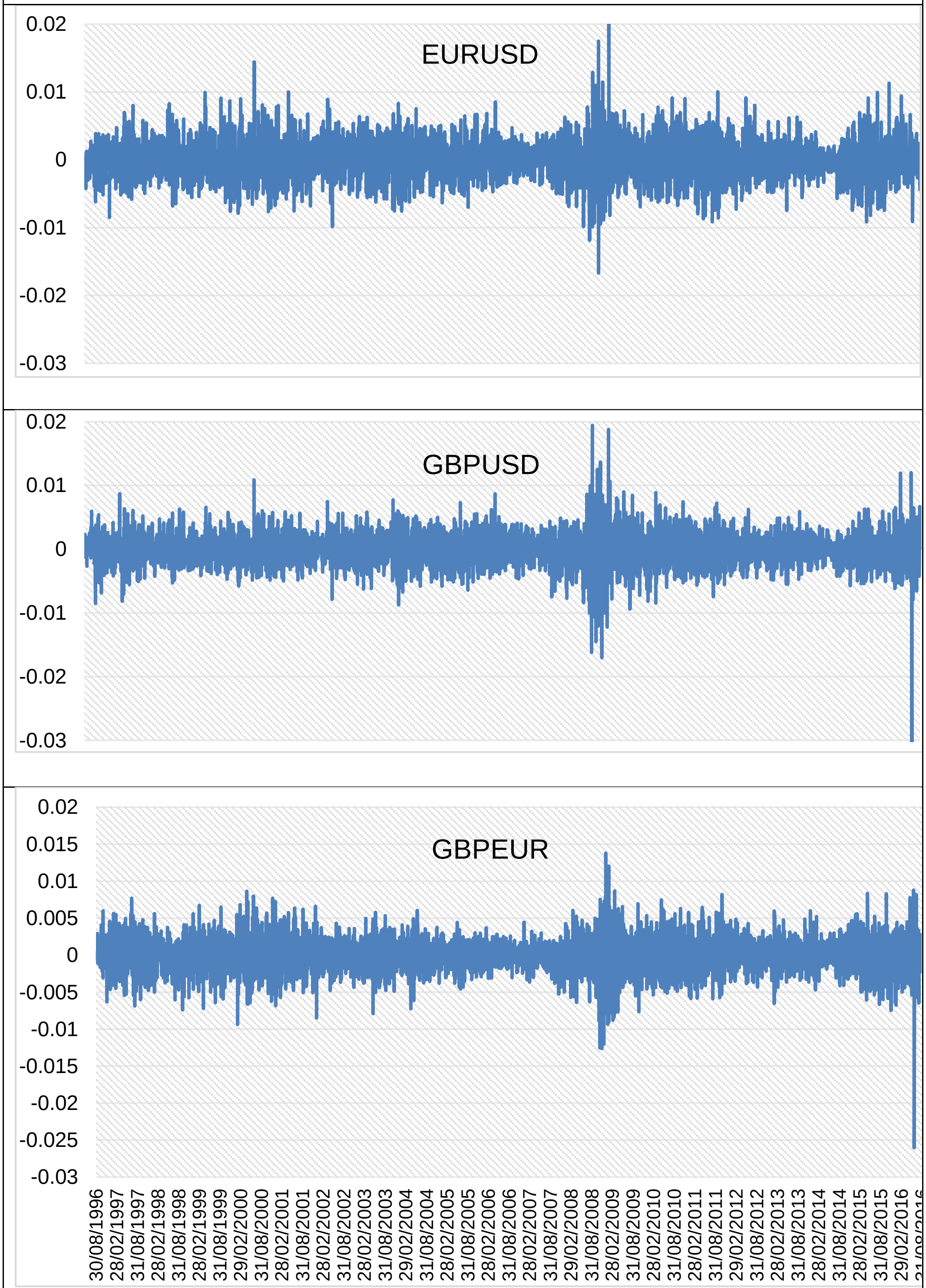
A plot of exchange rates returns of three currencies further emphasized the influence from the financial crisis. As Figure A.3.3.displays, during the period between September 2008 and the beginning of 2009 the volatility of all three exchange rates appear extremely volatile. And this volatility turmoil seems to first appear in the GBPUSD exchange rate, then EURUSD afterwards, and GBPEUR lastly.

**Figure A.3.2. EURUSD, GBPUSD, and GBPEUR from 1996 to 2016**




To examine the impact from the financial crisis to the volatility behaviours of exchange rate, an asymmetric component model was employed given its most explicit representation for short- and long-run volatility dynamics and volatility asymmetry property. The methodology used by Engle and Lee (1999) was also considered through this assessment since similar case is applied which studies the 1987's crash impact based on the component model. Two samples were used in the current analysis, with the first sample starting from 30/August/1996 to 01/September/2008, and the second sample from 30/August/1996 to 30/April/2009. The consideration here is that in the first sample the most turmoil period was excluded so as to isolate the major influence from the financial crisis. Comparing with the first sample with about 23 years long period observation, the second sample with only additional 8 month period is trivial. The present assessment examined any significant estimation difference between the two samples in view that the impact from financial crisis to the currency market is deemed to be dramatic.

**Figure A.3.3 Exchange Returns of three currency pairs**




The logarithmic returns of historical exchange rate between European Euro to United States Dollar (top), British Pound Sterling to US Dollar(middle), and British pound to European Euro (below) from 30/08/1986 until 30/08/2016 with 5219 observations.

**Table A.3.2 Asymmetric component GARCH model estimating results**

| | | $\beta$ | $\alpha$ | $\lambda$ | $\rho$ | $\phi$ | $\gamma_1$ | $\gamma_2$ |
|---|---|---|---|---|---|---|---|---|
| EURUSD | Prior-to-turmoil | 0.909 | 2.71E-05 | 0.099 | 0.995 | 0.025 | -488.132 | -0.004 |
| | | (4.74) | (0.50) | (0.78) | (395.28) | (3.95) | (-1.18) | (-0.01) |
| EURUSD | Turmoil period | 0.199 | 3.31E-10 | -0.126 | 0.996 | 0.029 | 1.230 | -1e-4 |
| | | (1.21E-03) | (3.13E-10) | (-0.04) | (33.14) | (0.20) | (0.01) | (-4.46E-05) |
| GBPUSD | Prior-to-turmoil | 0.919 | 3.97E-05 | 0.012 | 0.985 | 0.028 | -995.521 | 0.554 |
| | | (0.45) | (0.03) | (1.19E-03) | (227.01) | (0.03) | (-2.26) | (0.05) |
| GBPUSD | Turmoil period | 0.827 | 3.23E-05 | 0.086 | 0.992 | 0.036 | -999.631 | -0.103 |
| | | (1.60) | (0.37) | (0.19) | (180.89) | (2.14) | (-0.50) | (-0.05) |
| GBPEUR | Prior-to-turmoil | 0.311 | 0.062 | 0.095 | 0.997 | 0.029 | -0.511 | -0.012 |
| | | (13.01) | (2.78) | (5.39) | (473.60) | (5.64) | (-17.83) | (-1.08) |
| GBPEUR | Turmoil period | 0.056 | 0.065 | 0.007 | 0.996 | 0.036 | -0.394 | -0.005 |
| | | (28.73) | (21.50) | (8.00) | (200.72) | (9.72) | (-79.91) | (-1.18) |

Notes: Asymmetric component GARCH Model estimating results based on the prior-to-turmoil period sample and the turmoil period sample. Model coefficients are introduced in Chapter.4 Table 4.3. For the prior-to-turmoil period the sample starts from 30/August/1996 to 01/September/2008 while for the turmoil period the sample starts from 30/August/1996 to 30/April/2009.

Table A.3.2 reports the estimating results before and after the volatility turmoil during the financial crisis. As the results illustrate, for all three exchange rates the information shock has an increased impact to the volatility dynamic over the long run during the volatility turmoil, which is reflected in the $\phi$ s in the long run volatility equation. Furthermore, for both EURUSD and GBPUSD the short run volatility asymmetry parameters $\gamma_1$ exhibit fairly large estimated values, but in the long run this volatility asymmetry appear much less pronounced. This impact is further supported by the $\gamma_1$ in GBPEUR estimating results, which is more significant during the turmoil stage. In comparison, the long run volatility asymmetry parameter $\gamma_2$ does not indicate any significant effect which corroborates the earlier analysis of the foreign exchange volatility symmetry during the long run. Lastly, the sum of $\alpha$ and $\beta$ which represents the volatility mean-reversion rate during the short run, yields the reduced values from the prior-to-turmoil stage to the turmoil period across all three exchange rates, which is consistent with our observation of the volatile exchange rate period during the turmoil.

Lastly, the current assessment examined the impact of the financial crisis to the canola commodity market and to determine if any significant difference arose before and after the turmoil time. Figure A.3.4 and Figure A.3.5 display the price level and price change of the canola spot and futures market over our sample period from 2003 to 2016. As Figure A.3.4 indicates, during the financial crisis 2007-2008 there is significant price increase in both the canola spot and futures markets. Unlike the equity market, price increase is deemed as bad news in the commodity market and afterwards there is an accompanied volatility increase around 2008.

## Figure A.3.4 Canola spot and futures price 2003-2016


## Figure A.3.5 Canola spot and futures price returns

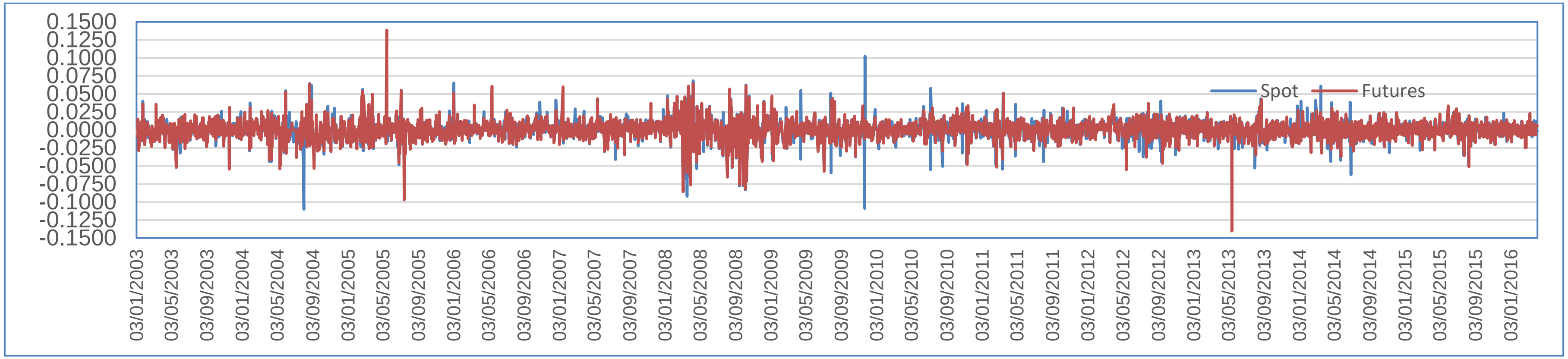


**Table A.3.3 The estimated BEKK models before and during the financial crisis**

$$\sigma_{s,t}^2 = c_{s,t-1} + A_{11}^2\varepsilon_{s,t-1}^2 + 2A_{11}A_{21}\varepsilon_{s,t-1}\varepsilon_{f,t-1} + A_{21}^2\varepsilon_{f,t-1}^2 + B_{11}^2\sigma_{s,t-1}^2 + 2B_{11}B_{21}\sigma_{s,t-1}\sigma_{f,t-1} + B_{21}^2\sigma_{f,t-1}^2 + G_{11}^2\varsigma_{s,t-1}^2 + 2G_{11}G_{21}\varsigma_{s,t-1}\varsigma_{f,t-1} + G_{21}^2\varsigma_{f,t-1}^2,$$

$$\sigma_{f,t}^2 = c_{f,t-1} + A_{12}^2\varepsilon_{s,t-1}^2 + 2A_{12}A_{22}\varepsilon_{s,t-1}\varepsilon_{f,t-1} + A_{22}^2\varepsilon_{f,t-1}^2 + B_{12}^2\sigma_{s,t-1}^2 + 2B_{12}B_{22}\sigma_{s,t-1}\sigma_{f,t-1} + B_{22}^2\sigma_{f,t-1}^2 + G_{12}^2\varsigma_{s,t-1}^2 + 2G_{12}G_{22}\varsigma_{s,t-1}\varsigma_{f,t-1} + G_{22}^2\varsigma_{f,t-1}^2;$$

| | $c_{s,t-1}$ | $c_{f,t-1}$ | $A_{11}$ | $A_{12}$ | $A_{21}$ | $A_{22}$ | $G_{11}$ | $G_{12}$ | $G_{21}$ | $G_{22}$ | $B_{11}$ | $B_{12}$ | $B_{21}$ | $B_{22}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Before the Crisis* | | | | | | | | | | | | | | |
| Estimates | 0.003 | 0.002 | 0.000 | 0.017 | -0.003 | 0.003 | 0.121 | -0.020 | 0.005 | 0.008 | 0.962 | 0.002 | -0.011 | 0.972 |
| T-statistic | (2.63) | (1.77) | (0.88) | (1.08) | (-0.31) | (0.30) | (1.46) | (-0.32) | (0.40) | (0.14) | (45.12) | (0.40) | (-0.44) | (42.61) |
| *During the crisis* | | | | | | | | | | | | | | |
| Estimates | 0.002 | 0.004 | 0.365 | -0.037 | -0.001 | 0.362 | 0.029 | 0.038 | 0.066 | 0.000 | 0.876 | -0.013 | 0.029 | 0.856 |
| T-statistic | (1.34) | (2.42) | (3.53) | (-0.27) | (-0.15) | (6.65) | (0.36) | (0.37) | (1.27) | (0.23) | (9.93) | (-0.20) | (0.42) | (11.25) |

Note: BEKK estimating results of canola spot and futures market price change and volatility for the before-the-crisis period and during-the-crisis period. The former period starts from 01/Aug/2003 to 30/August/2007 and the latter period starts from 03/Sep/2007 to 30/Dec/2011. $c_{1,1}$ and $c_{2,2}$ are the model intercepts, $A_{11}$ and $A_{22}$ describe the own-information shock effect of canola spot and futures volatility series. $A_{12}$ and $A_{21}$ indicate the arrival of information shock across markets. $B_{11}$ and $B_{22}$ measure the volatility persistence. $B_{21}$ and $B_{12}$ describe the volatility spillover magnitude. $G_{1,1}$ , $G_{1,2}$ , $G_{2,1}$ and $G_{2,2}$ are parameters capturing the asymmetry of volatility evolution. $\varsigma_{i,t-1} = \varepsilon_{i,t-1} I_{\varepsilon_{i,t-1}<0}$ with $I_{\varepsilon_{i,t-1}<0}$ the indicator function which has value of one when $\varepsilon_{i,t-1} < 0$, '***','**','*', denote 0.1%, 1%, and 5% significance level, respectively. The maximum likelihood estimation approach is used during the parameter estimation.

Table A.3.3 reports the estimating results of the asymmetric BEKK GARCH model, in which the price change and volatility evolution of the two markets were examined. As the table illustrates, during the period before the crisis, both spot and futures market did not exhibit any pronounced price change and volatility variation. The statistically significant parameter $G_{11}$ suggests the asymmetric return -volatility relationship in the spot markets but this interrelation does not apply in the futures. The volatility persistence parameters $B_{11}$ for spot market volatility and $B_{22}$ for the futures market are both statistically significant for periods before- and during-crisis. However, the estimated values of $B_{11}$ and $B_{22}$ show the reduced values during the crisis time, with the $B_{11}$ declining from 0.962 to 0.876, and with $B_{22}$ declining from 0.972 to 0.856. Their decreased values indicate that the financial crisis is more transient than other periods. Furthermore, during the crisis period, the estimated $A_{11}$ and $A_{22}$ both exhibit higher and statistically significant values, which further emphasize the severity of the financial crisis. Lastly, the significant parameter of $B_{22}$ during the crisis period suggests that the negative returns in the spot market partly arises from the futures market.

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